DOCUMENT RESUME

ED 069 527 SE 015 338

AUTHOR	Crook, Edwin F.
TITLE	Equa-formu-alities (Equations - Formulas - Inequalities).
INSTITUTION	Oakland County Schools, Pontiac, Mich.
SPONS AGENCY	Bureau of Elementary and Secondary Education (DHEW/OE), Washington, D.C.
PUB DATE	Nov 70
GRANT	OEG-68-05635
NOTE	78p.; Revised Edition
EDRS PRICE	MF-$0.65 HC-$3.29
DESCRIPTORS	Curriculum; Graphs; *Inequalities; Instruction; *Instructional Materials; Low Ability Students; Mathematics Education; Objectives; *Secondary School Mathematics; Units of Study (Subject Fields); Worksheets
IDENTIFIERS	Equations; ESEA Title III; Formulas Mathematics

ABSTRACT

This instructional unit is designed to serve as an introduction to algebra. True and false mathematical sentences are first presented with open sentences to introduce the use of a variable. Inequalities, formulas, and graphs are the next major concepts discussed. The unit concludes with six projects that attempt to tie the major concepts together. A teacher's guide is also available. Related documents are SE 015 334 - SE 015 337 and SE 015 339 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

ED 069527

EQUA•FORMU•ALITIES

U.S. DEPARTMENT OF HEALTH
EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

SE 015 338

OAKLAND COUNTY MATHEMATICS PROJECT STAFF

Dr. Albert P. Shulte, Project Director
Mr. Terrence G. Coburn, Assistant Director and Project Writer
Dr. David W. Wells, Project Coordinator
Mr. Stuart A. Choate, Project Writer
Mr. Philip L. Cox, Project Writer
Mr. Daniel L. Herman, Project Writer
Mr. Robert E. Peterson, Project Writer
Miss Diane M. Ptak, Project Writer
Miss Joyce Sweet, Project Writer
Mrs. Kathleen Danielson, Special Project Writer
Mrs. Lawanda Beane, Secretary
Miss Shirley Biggs, Secretary
Miss Deborah J. Davis, Secretary
Mrs. Judy Orr, Secretary
Miss Linda Tee, Secretary

ED 069527

EQUA•FORMU•ALITIES

(EQUATIONS - FORMULAS - INEQUALITIES)

OAKLAND COUNTY MATHEMATICS PROJECT

All units have benefited from the combined attention of the entire project staff. The major writing responsibility for this unit was handled by

EDWIN F. CROOK

ILLUSTRATIONS BY KATHLEEN DANIELSON

Oakland Schools
2100 Pontiac Lake Road
Pontiac, Michigan 48054

REVISED EDITION — NOVEMBER, 1970

This unit was prepared under the auspices of U.S.O.E. Grant 68–05635–0, Title III, E.S.E.A.

TABLE OF CONTENTS

Title	Page
Lesson 1 - TRUE OR FALSE	1
Lesson 2 - NOT ALWAYS TRUE OR FALSE	5
Lesson 3 - WELL BALANCED SENTENCES	9
Lesson 4 - NOT SO WELL BALANCED SENTENCES . .	17
Lesson 5 - BALANCED SENTENCES NOT ON THE BEAM . .	28
Lesson 6 - OPERATING ON BALANCED SENTENCES . . .	35
Lesson 7 - OPERATING ON UNBALANCED SENTENCES . .	41
Lesson 8 - SOLUTION PICTURES	45
Lesson 9 - PHONE CALL GRAPHS	51
Lesson 10 - WRITING MATHEMATICAL SENTENCES . . .	55
Lesson 11 - USING MATHEMATICAL SENTENCES	61

PREFACE

Equaformualities stands for a combination of equations, formulas, and inequalities. This is what this book is all about. Equations, formulas, and inequalities make up the basic mathematical language used by engineers, technicians, and many other people who use mathematics in their work or in their hobbies. For example, in drag racing formulas are used to place the cars in the various classes. In baseball, formulas are used to figure batting averages and pitchers' earned run averages. Inequalities are used in figuring weight classes in many sports, such as wrestling, boxing, auto racing, and boat racing.

There is nothing mysterious about equations, formulas, or inequalities. They are simply a shorter way of writing mathematical ideas or mathematical rules. They are sometimes called a shorthand way of writing, using a mathematical language.

This book will give you a chance to work with this mathematical language. Some of the basic operations in the language are introduced. You will use these operations in laboratory experiments and in several types of formulas.

TRUE OR FALSE

Study the following sentences.

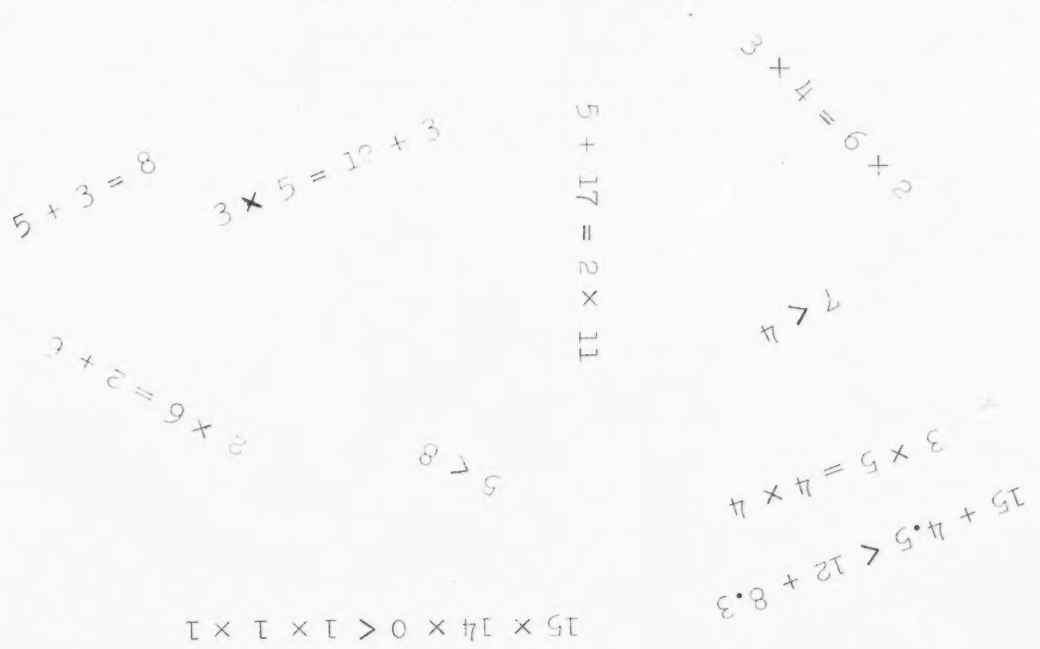
1. Lansing is the capital of Michigan.
2. New York is the capital of England.
3. Students should have 25 hours of homework each night.
4. The moon is a planet.
5. The Statue of Liberty is a bridge.
6. Hawaii is a state.

DISCUSSION QUESTIONS

Which of these sentences are true? Which are false? How can you tell which are true?

Numbers can also be placed in sentences. Here are some examples of mathematical sentences.

$$2 + 8 = 4 + 4$$



THOUGHT QUESTIONS

On page 1, some of the English sentences are true and some are false. Some of the mathematical sentences at the bottom of page 1 are true and some of them are false.

1) Which of the mathematical sentences are true and which are false?
2) How can you tell if one of the mathematical sentences is true?

EXERCISES

In the exercises below, you are to: (1) replace the ☐ with the number indicated; (2) state whether the resulting sentence is true or false. Examples A, B, and C have been worked for you.

	Sentence	☐ = 5	☐ = 7	☐ = 2
A.	☐ + 5 = 12	False	True	False
B.	☐ + ☐ = 2 × ☐	True	True	True
C.	☐ + 5 = 11	False	False	False
1.	☐ + 3 = 8			
2.	☐ × 4 = 28			
3.	1 + ☐ + 3 = 4 + ☐			
4.	13 + ☐ = ☐ + 13			
5.	54 - ☐ = 49			
6.	35 ÷ ☐ = 5			

EXERCISES (Continued)

	Sentence	□ = 5	□ = 7	□ = 2
7.	$5 + (\square + 3) = (5 + \square) + 3$			
8.	$\square = 1$			
9.	$\square \times 7 = 39$			
10.	$\frac{\square}{\square} \times 0 = 0$			
11.	$\square + \square + \square + 7 =$ $\square + \square + \square + \square$			
12.	$5 \times (\square + 8) =$ $(5 \times \square) + (5 \times 8)$			
13.	$\square + 5 = \square + 7$			

DISCUSSION QUESTIONS

1. Which exercises were true for all three values substituted?

2. Would these sentences be true for <u>all</u> number replacements of the □?

3. Which sentences were true for the first two values and false for the third?

4. Would any of the sentences in (1) be true for values of the □ other than 5, 7, or 2?

5. Which sentences were true for only one given value of the □?

DISCUSSION QUESTIONS (Continued)

6. Would any of the sentences in (5) be true for any other value of the $\square$?

7. Which sentences were false for all of the values substituted?

8. Which of the sentences in (7) would be true for some substitution for the $\square$?

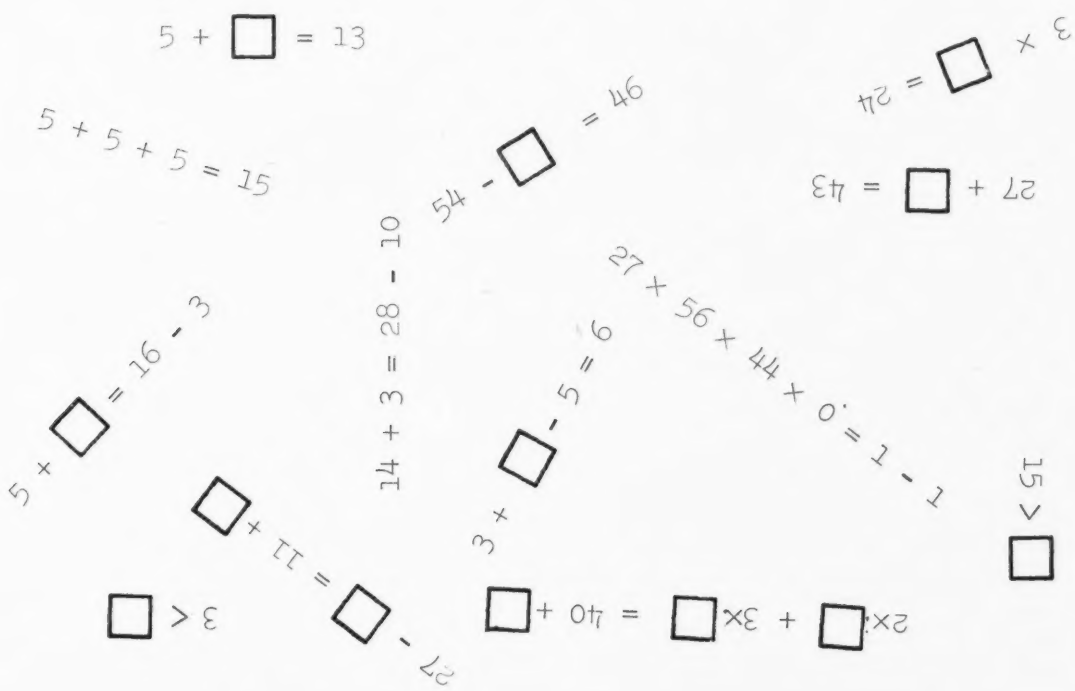
9. Were any of the sentences examples of the commutative, associative, or distributive laws? If so, which ones?

NOT ALWAYS TRUE OR FALSE

1. The U.S. Supreme Court is elected by the people of Washington, D.C.
2. He is governor of this state.
3. The President of the U.S. is elected every four years.
4. That one is your favorite.
5. He is a real winner.

DISCUSSION QUESTIONS

1. Which of the sentences above are true? _____
2. Which of these are false? _____
3. Which ones are neither true nor false? _____
4. Why can't you tell if some of these sentences are true or false?



DISCUSSION QUESTIONS (Continued)

5. Which of the mathematical sentences are true? Which are false?
6. Which of these mathematical sentences are neither true nor false?
7. If $\square$ is replaced by 10 in each sentence, could you tell if the sentence is true or false?
8. Which ones would be true if the $\square$ was replaced with the number 10?
9. In each sentence, what number does the $\square$ have to be replaced by to get a true sentence?

When we have a mathematical sentence such as $5 + \square = 14$ we call the $\square$ a variable. A sentence containing a variable is neither true nor false. When we substitute numbers for the variables, we get either a true sentence or a false sentence. We are primarily interested in those values which make the sentence true. These values are called the solution set of that sentence.

In $5 + \square = 14$ what number could replace the $\square$ to make a true sentence?

Right! A 9 in place of the $\square$ would make a true sentence because $5 + 9 = 14$.

In the sentence $2 \times \square + 3 = 11$, if the $\square$ could be replaced by any member of the set $\{1, 2, 3, 4, 5\}$, which number(s) would make the sentence true?

EXERCISES

In the following mathematical sentences you are to find the value(s) for the variables (□'s, △'s or ○'s) that make the sentence true. All the values are to be selected from { 0, 1, 2, 3, 4, 5}.

1. □ + 7 = 11
2. 3 + △ + 2 = 10
3. 3×□ > 9
4. 4×□ + 7 = 23
5. 5×□ = 30
6. 3×□ < 5
7. 6 - □ = □ - 2
8. □ - 19 = 2 - 2
9. 45 > 47 - □
10. □ + 7 = 7 + □
11. △ + △ = △ × △
12. 3×□ + 15 = 27

The set from which we choose replacements for a variable is known as the replacement set. In the following exercises, use the replacement set {0, 1, 2, 3, 4, 5, $5\frac{1}{2}$, 6, 7, 8, 9, 10}. For each sentence, give all the values that make the sentence true.

1. □ < 14 - 8
2. □ × 1 = □
3. 2×(□×3) = (2×□)×3
4. □ : □ < 2
5. 2×□ + 6 = 17

6. $\triangle + \triangle + \triangle = 3 \times \triangle$

7. $4 \times \square + 6 = 12$

8. $\frac{\square}{4} = 17 - 17$

9. $4 \times (\square + 2) = (4 \times \square) + (4 \times 2)$

10. $\frac{\square}{2} + \square = 12$

11. $3 \times \triangle > 25$

12. $4 \times \square + 1 = 18$

See if you can find the values of the variables that make these sentences true. Use only names of whole numbers to replace the variables (except in Exercise 1).

EXERCISES

1. $\square$ is governor of Michigan

2. $\triangle + \triangle + \triangle + \triangle = 45$

3. $\square + 5 = \square - 4$

4. $\square + 3 < 4$

5. $\square + 5 > \square + 5$

6. $(3 \times \triangle) + 1 = 19$

7. $\triangle + \triangle + \triangle = (2 \times \triangle) + \triangle$

8. $\frac{\square}{\square} < 3$

WELL BALANCED SENTENCES

BALANCE BEAM

A balance beam is used to weigh items. The beam will show which of two weights is heavier, or if they weigh the same.

How does a balance beam show that two objects have the same weight? Suppose you have a group of small blocks that all weigh the same. Place five blocks on one end of the balance beam and four blocks on the other end. Will the beam balance? _____ If not, which end will rise? _____

Because the end with five blocks is heavier, we can write $5 > 4$ or $4 < 5$. Add three blocks to the end with five blocks. Will the beam balance? _____

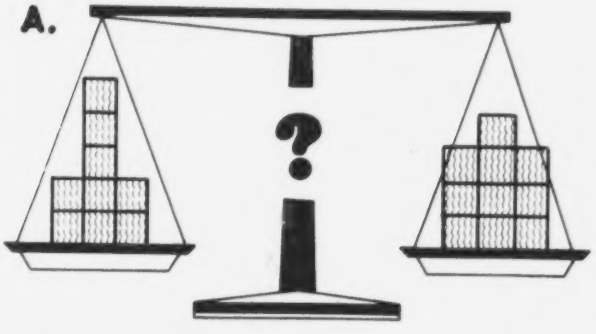
On the other end, add four blocks to the four already there. Will the beam balance now? _____

Now there are $5 + 3$ blocks on one end and $4 + 4$ on the other end. Make a mathematical sentence out of $5 + 3$ ____ $4 + 4$, by putting a $<$, $=$, or $>$ in the blank.

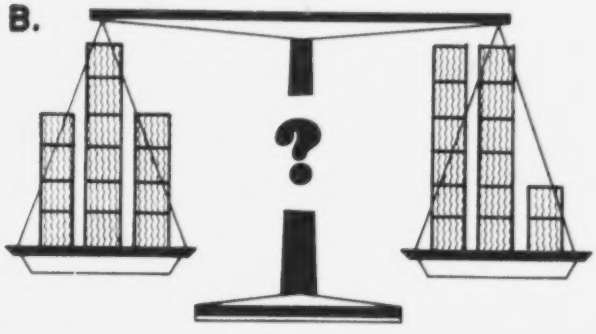
DISCUSSION QUESTIONS

In which of the following situations will the beam balance? Assume the small blocks all weigh the same.

A.



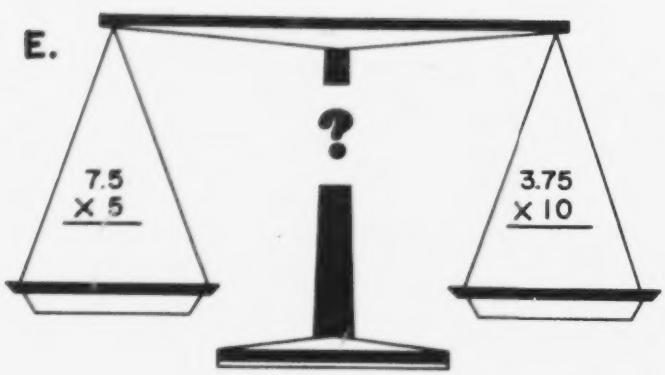
B.



C.

D.

E.

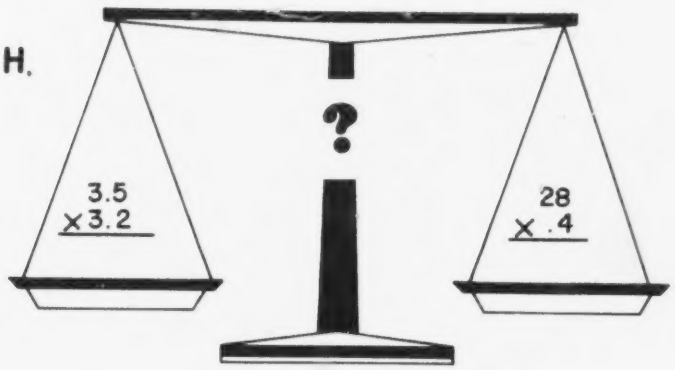


F.

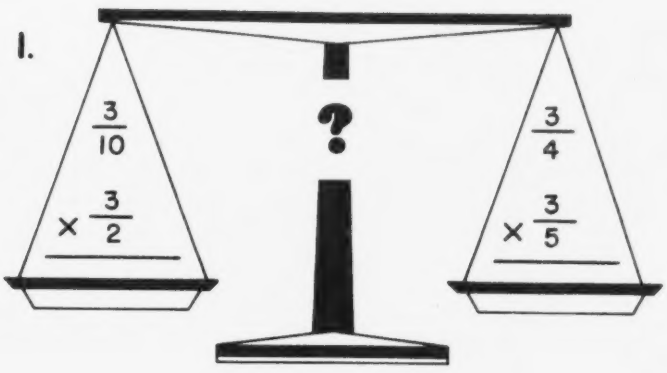
G.

4531
- 2644
?
4611
- 2734

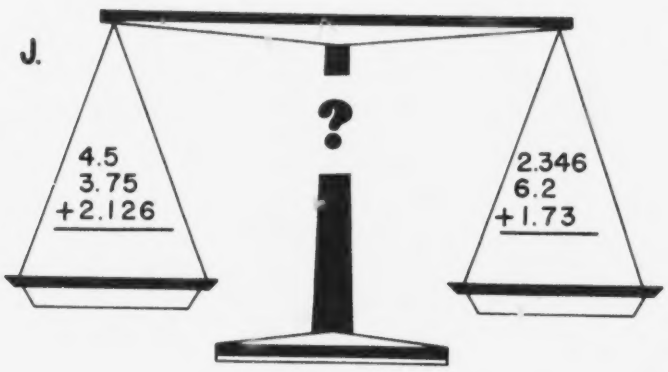
H.


3.5
× 3.2
?
28
× .4

I.

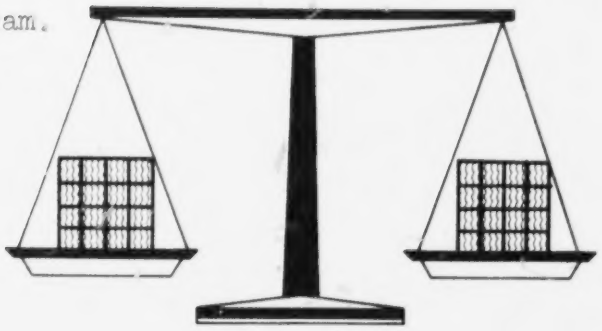

3/10
× 3/2
?
3/4
× 3/5

J.


4.5
3.75
+ 2.126
?
2.346
6.2
+ 1.73

EXERCISES

Let's look at this balance beam.



1. Does it balance?
2. If we take five blocks off each side, will it balance?
3. If we take $\frac{1}{2}$ the blocks off each side, will it balance?
4. If we double the number of blocks on each side, will it balance?
5. If we add ten blocks to each side, will it balance?
6. If we take away half the blocks on one side and take away nine blocks on the other side, will the scale balance?
7. What can you do to the weights and still have the beam balance?
8. What can't you do to the weights to keep the beam balanced?

Here is another beam:

We know the beam balances but we don't know how much the large block with the Δ on it weighs. How could we find the weight of the large block? Right again; if we take three small blocks off each side, the large block would be all by itself on one side and six small blocks would remain on the other side. Then we can say, "The large block weighs the same amount as six small blocks".

For each of these beams tell what you would have to do to find the weight of one of the large blocks.

EXAMPLES:

A.

REMOVE TWO BLOCKS FROM EACH SIDE.

B.

REMOVE HALF THE BLOCKS FROM EACH SIDE (OR DIVIDE THE NUMBER OF BLOCKS ON EACH SIDE BY TWO.)

EXERCISES

1.

2.

△ = _____

△ = _____

3.

_____

_____

_____

_____

△ = _____

4.

_____

_____

_____

_____

△ = _____

5.

_____

_____

_____

_____

△ = _____

What do you have to do in this problem to find the weight of one of the large blocks? This leads us to the decision as to which to do first; take away three small blocks or take half of each amount. If we take half of each side first, we would have to take half of the three small blocks on the left side. If we take away the three blocks first, we avoid this problem.

Thus, for this problem we would:

1. Take three small blocks away from each side.
2. Take half of the remaining blocks away from each side.

EXERCISES

Tell what you would do to find the weight of one large block in each of the balance scales. You should avoid removing parts of blocks if possible.

1.

_____

_____

_____

△ = _____

2.

_____

_____

_____

△ = _____

3.

_____

_____

_____

_____

△ = _____

4.

_____

_____

_____

_____

△ = _____

5.

_____

_____

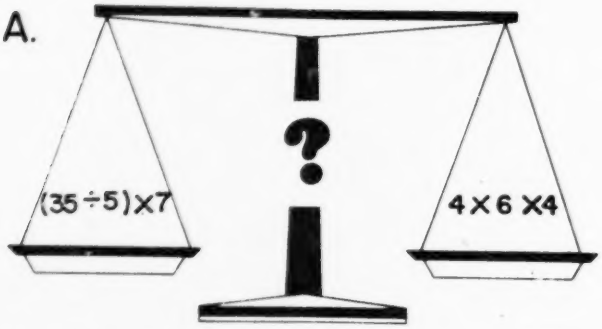
_____

_____

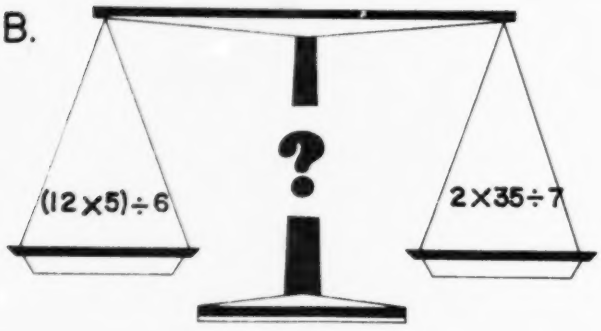
△ = _____

NOT SO WELL BALANCED SENTENCES

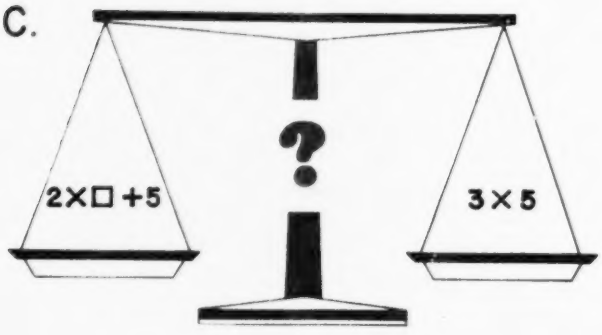
A.



B.



C.

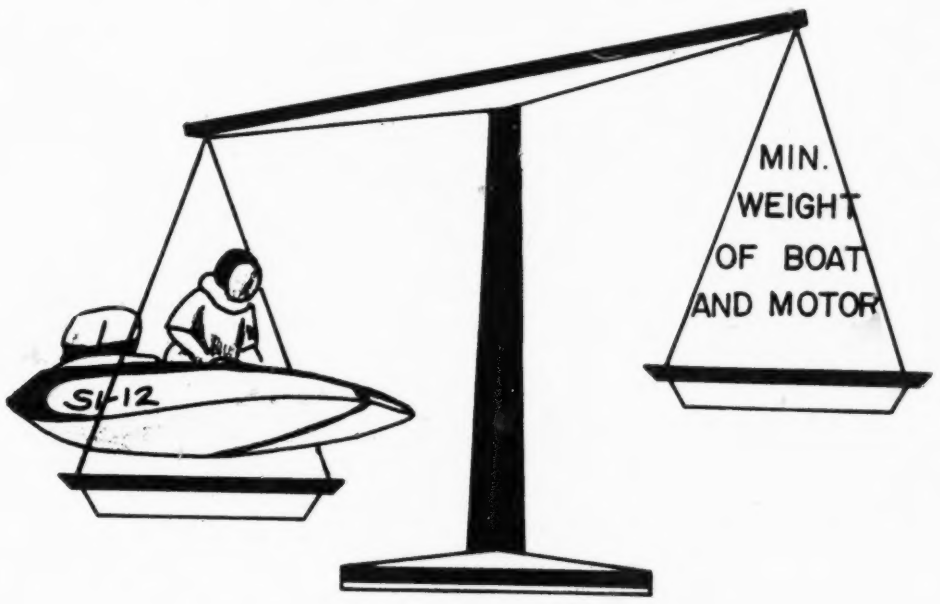


1. Which will balance? _____
2. Which will not balance? _____
3. For some of the equations above, it is impossible to tell whether or not they balance. Which?

In practically all cases so far the beams have been balanced. However, there are many situations where beams are unbalanced. For example, many things have to weigh more than a certain minimum or less than a certain maximum.

In many forms of automobile and boat racing, the car or boat must weigh at least a certain amount. When the car or boat is inspected by the race officials, tney weigh it. As long as it weighs more than the minimum weight, the inspectors are satisfied. The car or boat could weigh 1 lb., 100 lbs., or 500 lbs. more than the minimum and still meet the weight requirements.

Below is a picture showing the beam if the boat weighs more than the minimum weight requirements.



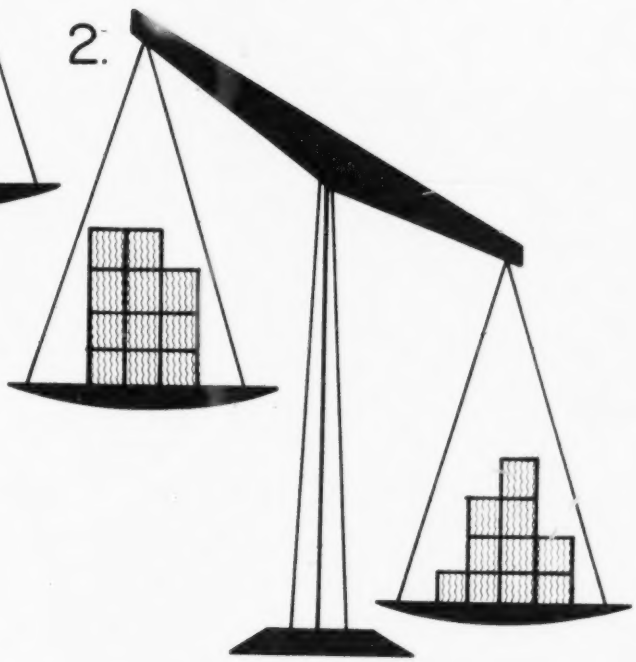
The American Power Boat Association rules state that a class DU Stock Outboard Hydroplane must weigh a minimum of 435 pounds. This weight includes the boat, motor, and driver.

EXERCISES

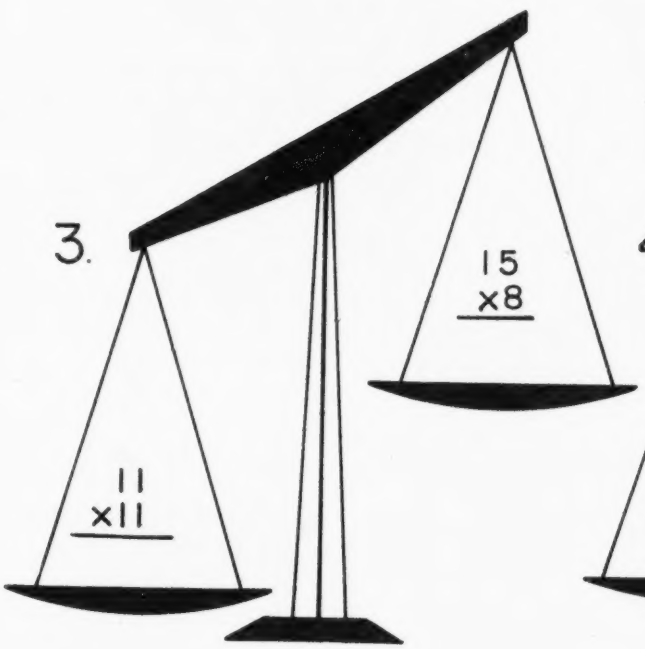
Which of these scales shows the balance tipped the correct way?

1.

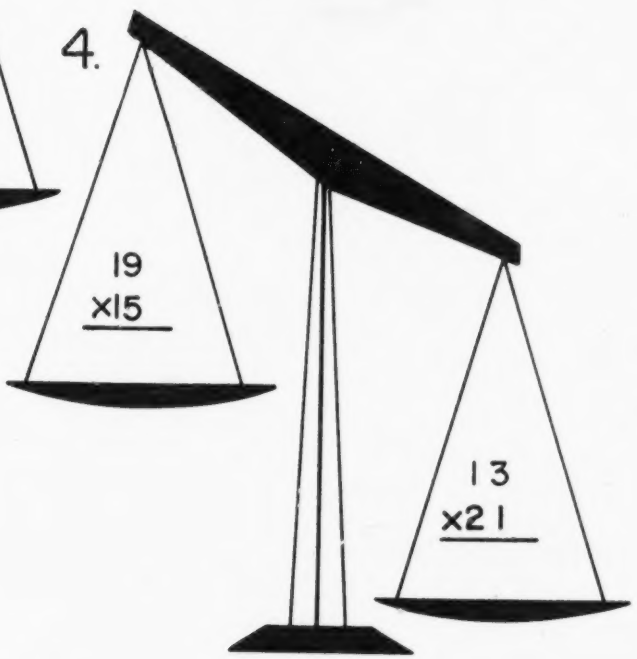
2.

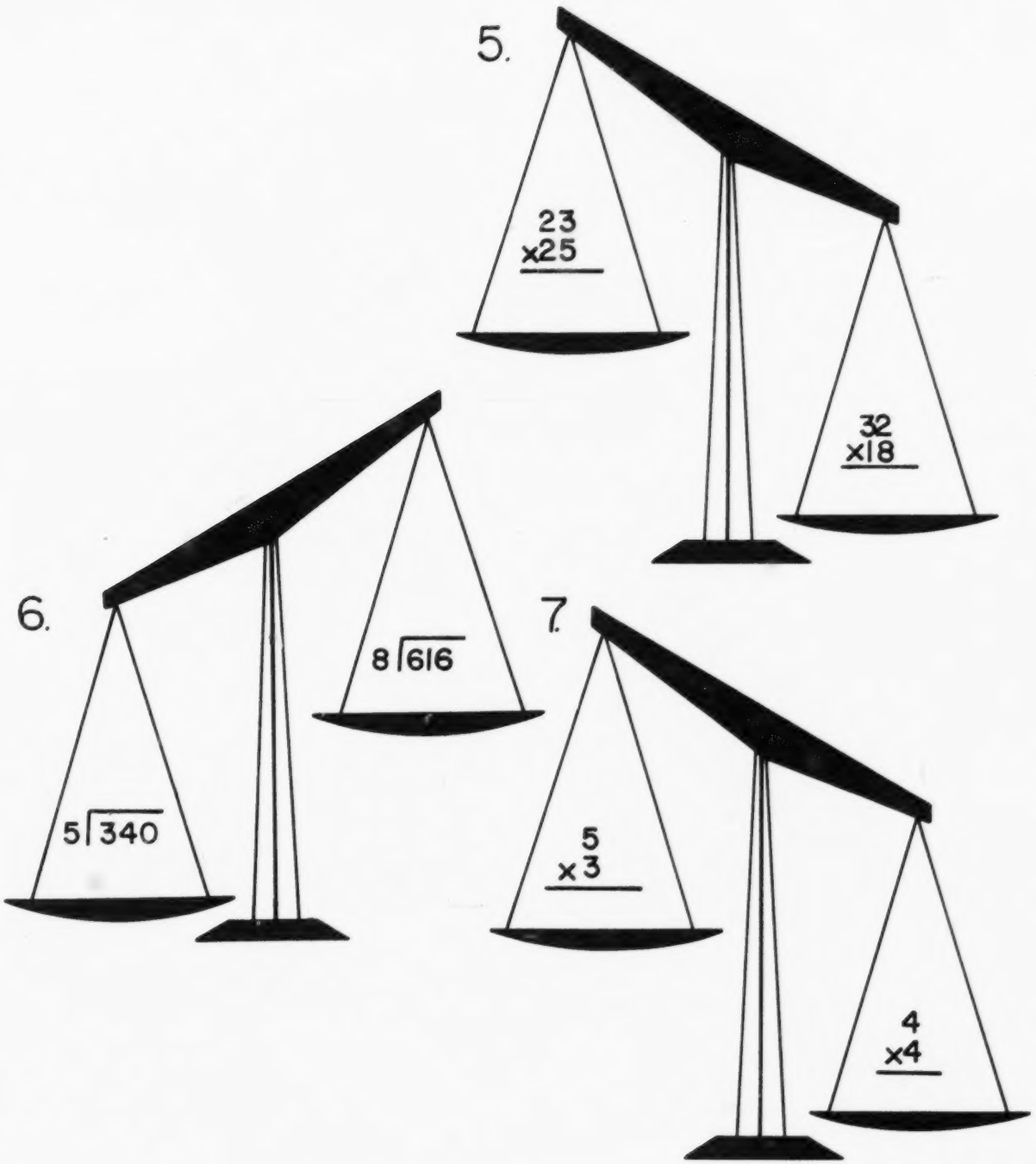


3.



4.

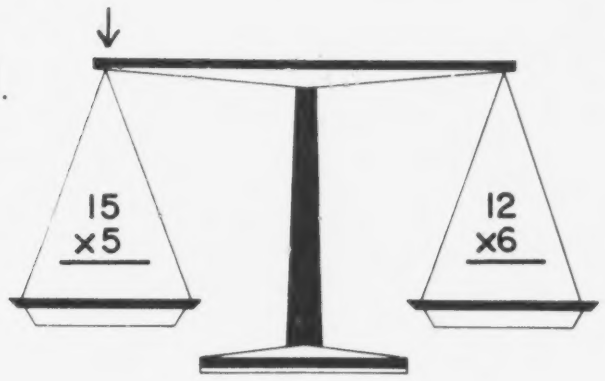




We can write this last relationship as a mathematical sentence:
$5 \times 3 < 4 \times 4$.

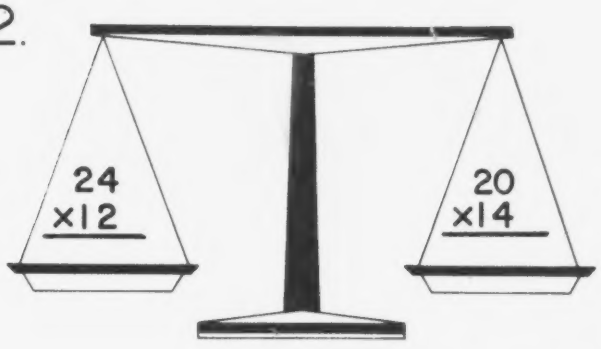
Write a mathematical sentence using =, < , or > signs for each of these balances. Also draw an arrow over the end of the beam that would move downward (as done in number 1).

1.



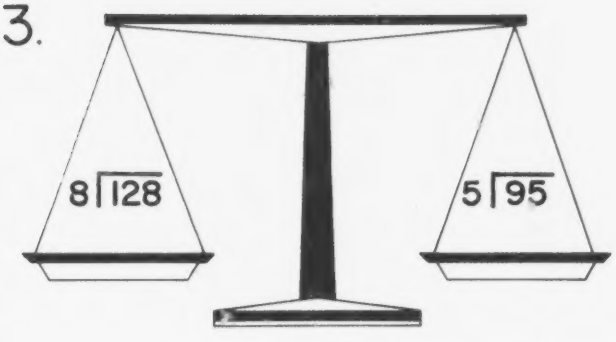
15 × 5 _____ 12 × 6

2.



24 × 12 _____

3.

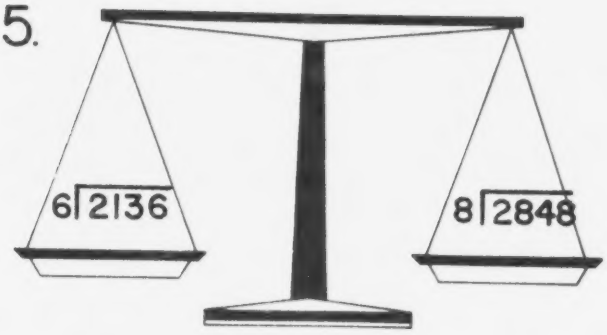


_____ _____

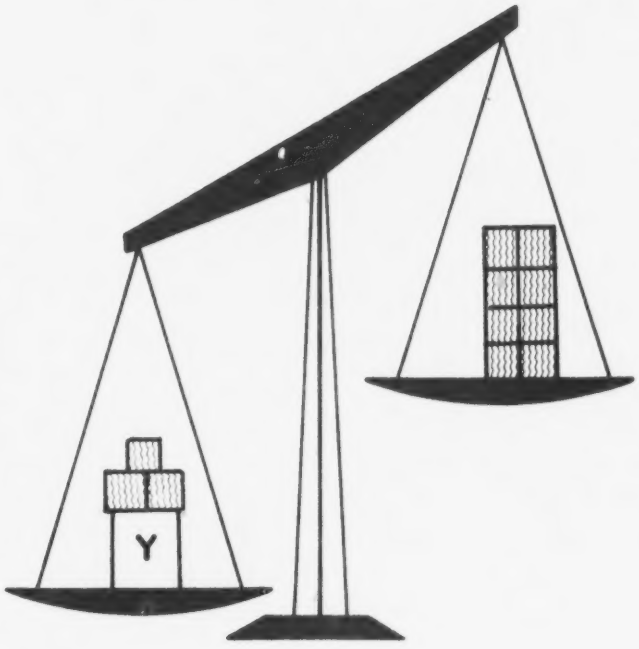
4.

_____ ___ _____

5.



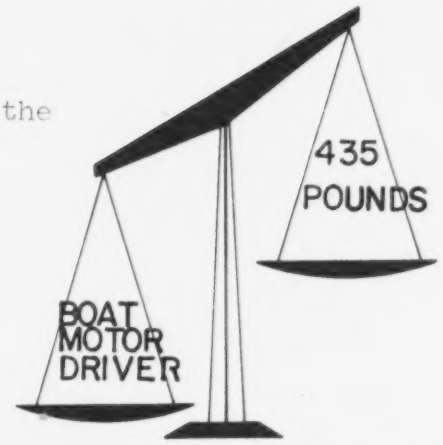
_____ ___ _____



From this drawing you can see that 1 large block plus 3 small blocks is heavier than 8 small blocks. A shorter way of writing this is: Y + 3× ▨ > 8 × ▨

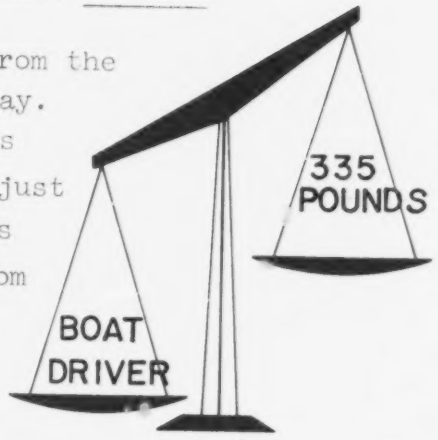
If we remove 3 small blocks from each side, will the beam still remain unbalanced _____? If it does, the beam will show 1 large block is heavier than _____small blocks, or Y > _____ × ▨ .

Let's consider the boat problem from page 16. As the problem stated the boat, motor, and driver must weigh more than 435 pounds.

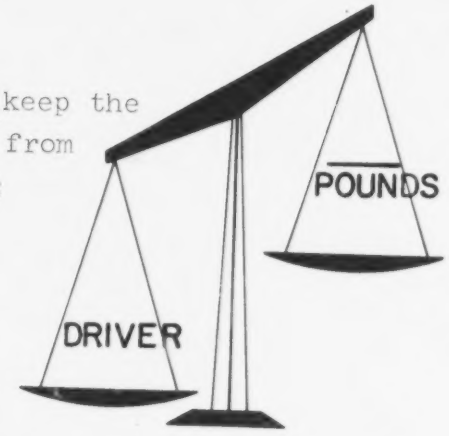


Does the combination of boat, motor, and driver above pass the weight requirement? _____

Now if the 100 lb. motor is removed from the scale, the beam would tip the other way. To prevent this, the 435 pound side is reduced until the beam is unbalanced just like it was originally. This requires 100 pounds of weight to be removed from the right side of the beam. Now the beam shows this:



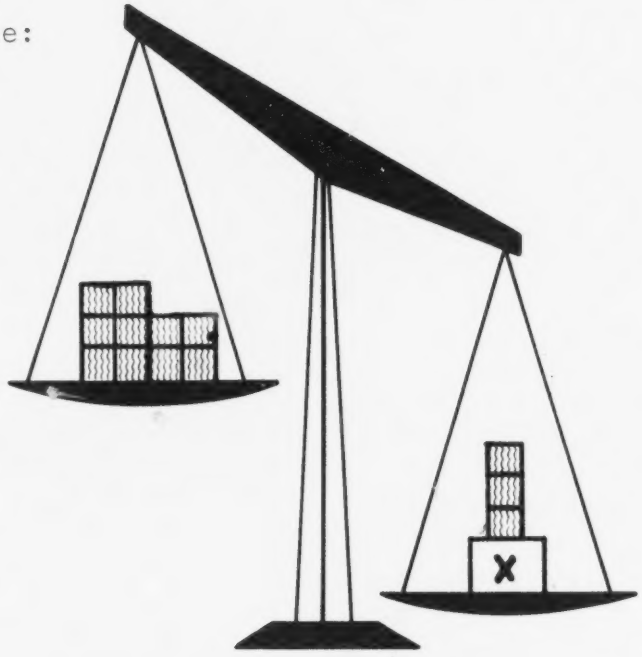
Now the 200 lb. boat is removed. To keep the beam the same, 200 pounds are removed from the other side. Now the scale shows:



In other words the driver must weigh > _____.

Let's look at another example:

Remove 3 small blocks from each side.



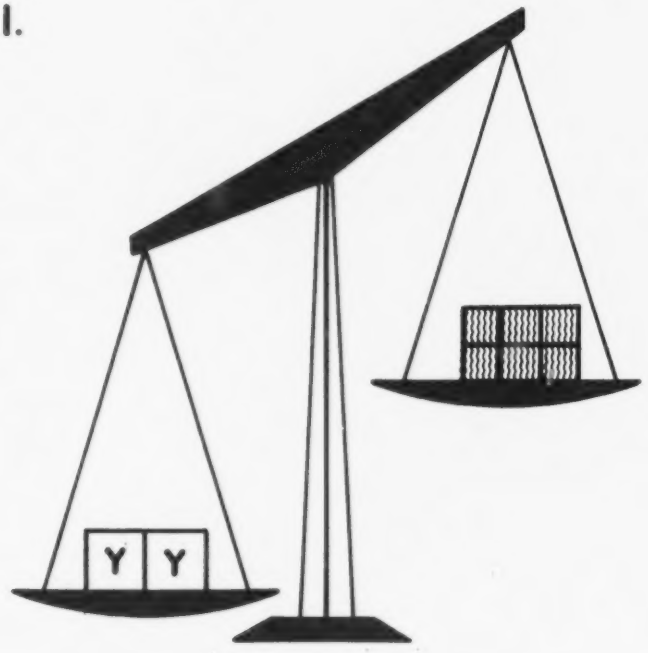
$10 \times \square < X + 3 \square$

$7 \times \square < X$

Thus: Seven small blocks weigh less than one large block.

Try these problems. In each exercise tell what you would do to find the weight of the large blocks.

1.

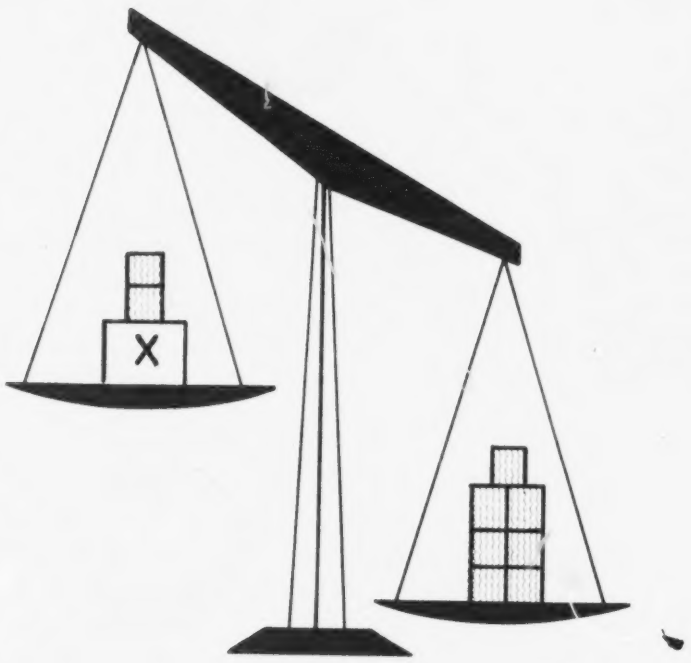


$2 \times Y > 6 \times \square$

$Y > _____$

_____

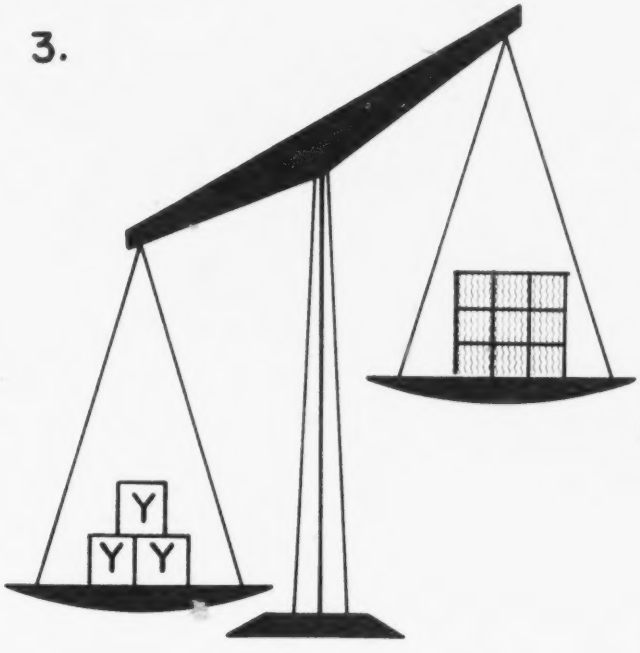
2.



X + 2 × □ < 7 × □

X < _____

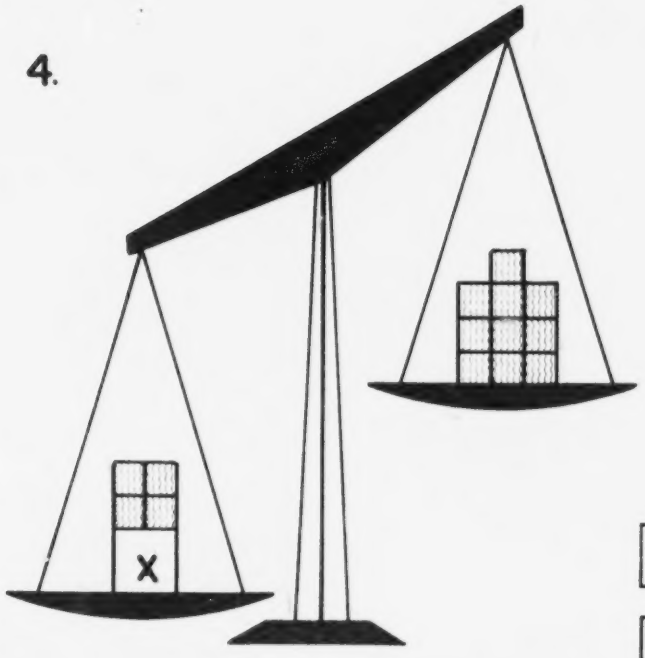
3.



3 × Y > _____ □

Y > _____ □

4.

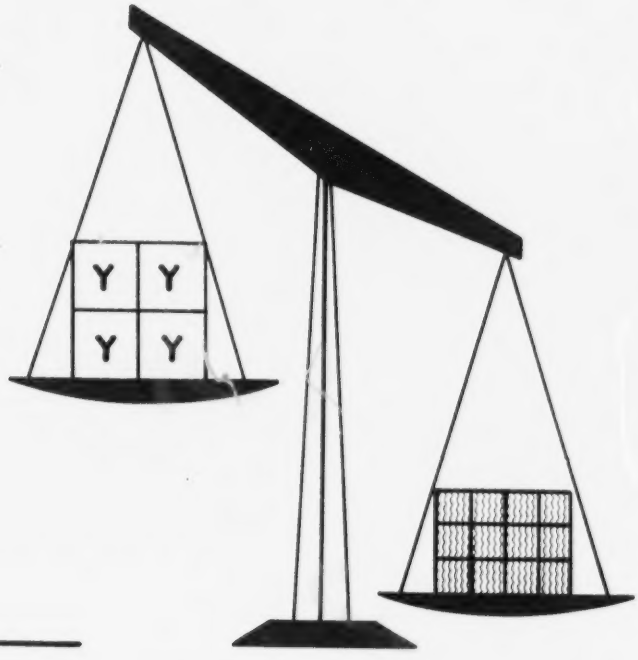


$\boxed{X} + 4 \times \square > 10 \times \square$

$\boxed{X} >$ _____

_____

5.

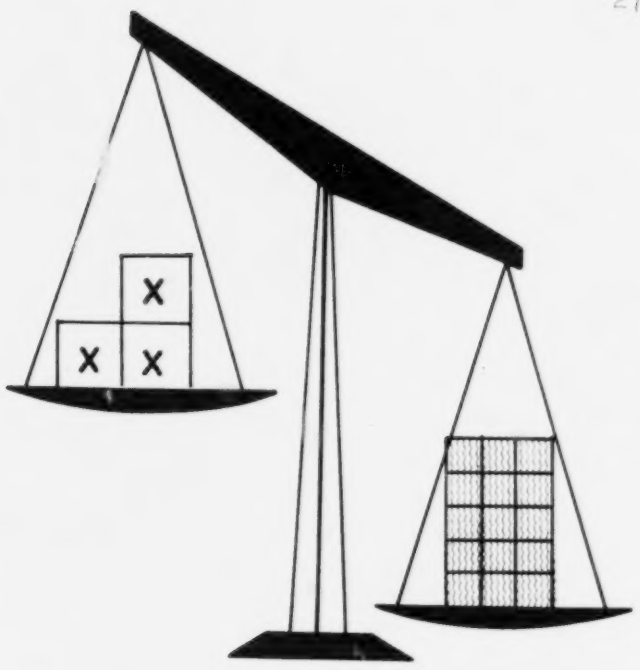


$4 \times \boxed{Y} < 12 \times \square$

$\boxed{Y} <$ _____

_____

6.

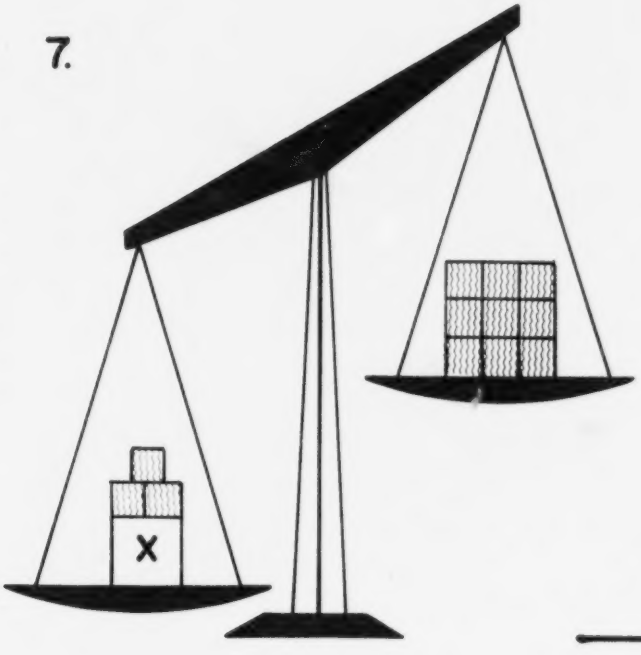


$3 \times \boxed{X} > 15 \times \square$

$\boxed{X} >$ _____

_____

7.



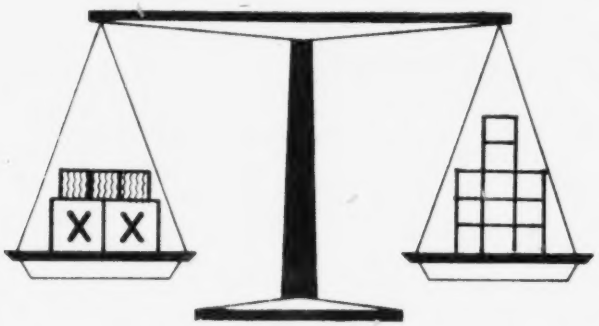
$\boxed{X} + 3 \times \square < 9 \times \square$

$\boxed{X} <$ _____

_____

BALANCED SENTENCES NOT ON THE BEAM

Equations and inequalities represented on a balance beam can be solved without using the beam. For example:



This would be written $2 \times \boxed{X} + 3 \times \blacksquare = 11 \times \blacksquare$.
However there is a much shorter way to write this mathematical sentence. In place of the unknown weights $\boxed{X}$ we can just write the X. Thus, the X will stand for the weight of the unknown box $\boxed{X}$. Thus our equation becomes

$$2 \times X + 3 \times \blacksquare = 11 \times \blacksquare$$

As you can see, there is a possibility of confusing the letter X and the multiplication sign. To prevent teachers from getting your answers confused, Algebra does away with the use of the multiplication sign $\times$ in many cases. When two symbols (not numerals) are written together without any sign between them, it means they are multiplied together. Thus $2\ell w$ means 2 times ℓ times w. This doesn't work when used between two numerals, of course. Does 45 mean 4×5? _____ Does $3\blacksquare$ mean $3 \times \blacksquare$? _____

Our equation becomes: $2X + 3\blacksquare = 11\blacksquare$.

Next we can make things even shorter by leaving out the $\blacksquare$ The number itself can stand for the number of blocks.

The equation is now simply: $2X + 3 = 11$

EXERCISES

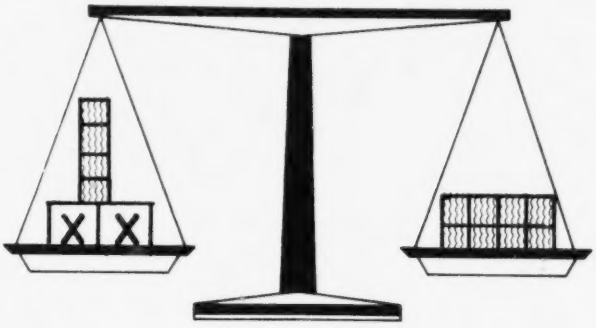
What is the simplest mathematical sentence you can write to represent this problem?

_____ X + _____ = _____

The answer is upside down at the bottom of the page.

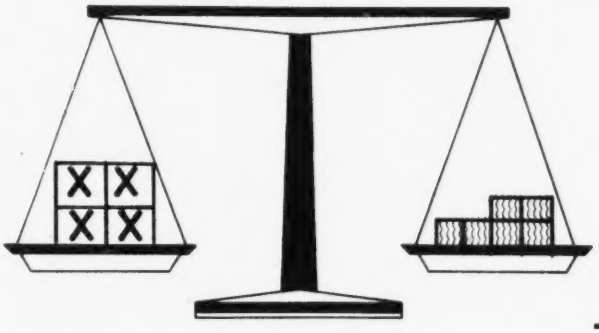
Do the same for the following exercises.

1.



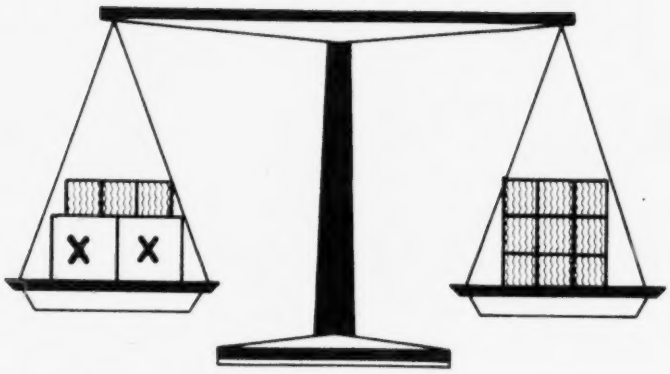
_____ + _____ _____

2.



_____ _____

$3X + 1 = 7$



A shorter way to write 1X is X.

Now consider the equation

$$2X + 3 = 9$$

a. Draw a balance beam to represent it.
b. Tell what you would do to the beam to end up with just the X on one side of the scale.

Now apply the same steps to the equation itself.

$$2X + 3 = 9$$

$$\underline{-\ 3 = -3} \qquad \text{Remove 3 from each side.}$$

$$2X = 6$$

$$\frac{2X}{2} = \frac{6}{2} \qquad \text{Divide each side by 2.}$$

$$X = 3 \qquad \text{This means } 1X = 3.$$

This means that X has the same weight or value as the 3. This process is known as solving the equation. Solving the equation is a fancy way of saying "Finding the value of the unknown in this equation".

EXERCISES

Draw X + 5 = 12 on a balance beam. Tell what you would do on the beam to find the weight of the X.
Now demonstrate the process with the equation.

$$X + 5 = 12$$

$$\underline{\qquad\qquad} \qquad \underline{\qquad\qquad\qquad}$$

$$X =$$

Example 1

Now consider $X + 135 = 163$

If you drew these blocks on the scale, what would you do to both sides to have the X remain by itself? _____

Now instead of drawing the beam, just do that step to the equation itself.

$X + 135 = 163$

$-\quad\quad -$

_____ _____

$X =$

Example 2

Consider $3X = 123$

On the balance beam, what would you do to both sides so that one side contained a single X? _____

$\underline{3X} = \underline{123}$

Do the same for the equation. _____

$X =$ _____

EXERCISES

Solve these equations: In each, write what you are going to do to both sides of the equation, then show this. Finally show the answer as $X = \underline{15}$ or $Y = \underline{27}$ rather than 15 or 27.

1. $X + 5 = 18$

$-\quad\quad = -$

_____ _____

$X =$ _____

2. $\underline{4X} = \underline{20}$

_____

$X =$ _____

3. $X + 17 = 49$

_____ _____ _____

$X =$ _____

4. $X + 145 = 355$

$X =$ _____ _____

5. $\underline{5X} = \underline{60}$

_____

$X =$ _____

6. $12X = 60$

_____

7. $X + 18 = 36$

_____

8. $X + 143 = 296$

_____

9. $2X + 15 = 41$

10. $3X + 8 = 44$

CHALLENGE EXERCISES

11. $4 + 10X = 64$

12. $3X + 7 = 12$

13. $129 = 8 + 11X$

OPERATING ON BALANCED SENTENCES

See if you can solve these two equations. In each, show what you do to both sides of the equation.

A. $5X = 25$

_____ _____

$X =$ ____

B. $5 + X = 25$

_____

$X =$ _____

DISCUSSION QUESTIONS

1. How do your answers compare?
2. If you got different answers, explain why this happened.
3. If you got the same answer both times, explain why this happened.

Solve these two equations:

C. $7 + X = 28$

_____

$X =$ _____

D.

$7X = 28$

_____

$X =$ _____

THOUGHT QUESTIONS

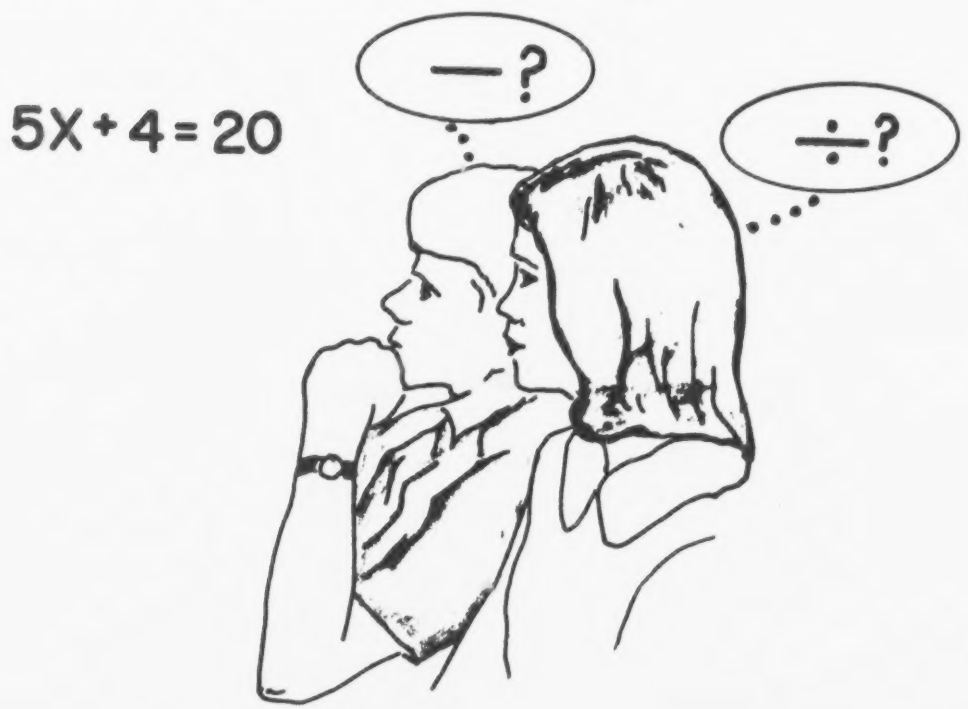
This time, did you get the same answer for parts C and D? _____ Why did this happen?

Did you use the same method to solve both equations?

If you used different methods to solve equations C and D explain why.

If you used the same method on both C and D explain why.

So far you have used two methods to solve equations; Subtraction and Division. Sometimes you subtracted the same number from both sides of the equation and other times you divided both sides of the equation by the same number.



Guessing which of the two methods to use doesn't work out too well because at most you could only expect to be right 50% of the time. There is an easier way to determine which method to use. Look at the variable (the letter) and determine what operations were done to it.

In 3+X=15 three was added to the X. To solve the equation you do the opposite. The opposite of adding 3 is subtracting 3.

In 3X=15 the X was multiplied by 3. The opposite of multiplying by 3 is dividing by 3. To solve the equation you must divide both sides of the equation by 3.

✓ **POINT**

Decide whether you would subtract or divide to solve each of these equations.

1. 3X = 25 _____
2. 102X = 2040 _____
3. 52 + X =75 _____
4. X + 76 = 1776 _____
5. 210X = 6300 _____ _____

✔ **POINT**

In each of the previous exercises, what number would you subtract from each side or divide both sides of the equation by?

1. _____
2. _____
3. _____
4. _____
5. _____

For $2X + 3 = 13$ do you divide or subtract? Think how you would solve this if it were represented on a balance beam. What would you do then? On the balance beam you first removed 3 blocks from each side. On the equation you do that operation first also. After you have removed everything but the X's from one side of the scale on the equation, then you divide to get a single X or variable.

Your work should show the solution as:

$$2X + 3 = 13$$
$$\underline{\quad - 3 = -3}$$
$$\frac{2X}{2} = \frac{10}{2}$$
$$X = 5$$

EXERCISES Solve these equations.

1. $3X + 7 = 37$
2. $12 + 5X = 22$
3. $1 + 4Y = 11$
4. $5Z = 37.5$

5. $X + 7 = 49$

6. $5X + 5 = 55$

7. $10X + 12 = 1042$

8. $15 + 12X = 159$

Some mathematical sentences have a special meaning: $60D = rt$ relates the speed, time and distance traveled by any moving object, such as a car or boat.

In $60D = rt$

D stands for the distance in miles.

r stands for the speed in miles per hour(mph).

t stands for the time in minutes.

If you know any two of the three quantities, you can find the third one.

9. For example, if a car travels 30mph for 10 minutes the equation $60D = rt$ becomes

$60D = 30 \times 10$ or

$60D = 300$. Find D.

Therefore a car traveling at 30 mph for 10 minutes travels _____miles.

10. If a boat travels 24 mph for 10 minutes, how far does it travel?

$60D = rt$

Using 24 in place of r and 10 in place of t, the equation becomes

$60D = 24 \times 10$

Multiplying 24×10 getting 240 gives

$60D = 240$. Find the value of D.

The boat traveled _____ miles.

Use the formula $60D = rt$ in solving the following exercises.

11. How many minutes will it take a car traveling at 30 miles an hour to go 10 miles?

12. If a boat is traveling at 15 miles an hour for 48 minutes, what distance will it have covered?

13. How fast is a plane traveling if it covers 40 miles in 4 minutes?

14. An elevator covers 52.8 feet vertically in $\frac{1}{2}$ minute. What is the rate in miles per hour that the elevator is traveling? (There are 5280 feet in one mile. D must be given in miles.)

OPERATING ON UNBALANCED SENTENCES

EXERCISES

Consider these two mathematical sentences: Draw a balance scale to represent each one.

A. $3 + X = 8$ B. $3 + X > 8$

THOUGHT QUESTIONS

How could you solve equations like A on the balance beam?
How could you solve inequalities like B on the balance beam?
How can you solve A without using the beam?
How would you solve B without using the beam?

EXERCISES

Solve these:

C. $3X = 16$ D. $3X < 16$

Describe the step you used to solve each mathematical sentence.

Solve these two exercises:

E. $3X + 5 = 17$ F. $3X + 5 > 17$

Explain any differences in the way you solve these two exercises.

EXERCISES Solve these mathematical sentences.

1. $7X = 91$

2. $X + 7 > 91$

3. $X + 3.75 = 42.85$

4. $3.8X < 57$

5. $3X + 5 > 15.5$

6. $10X + 17 = 85$

$d = 16t^2$ is a formula for the distance a falling object travels in a given number of seconds.

d stands for the distance in feet.

t stands for the number of seconds.

If a stone is dropped from a cliff and falls for three seconds before it hits the ground, how high is the cliff?

$d = 16t^2$

Of course you remember t^2 means $t \times t$, so if t is three seconds then t^2 is 3×3 or 9. Thus: $d = 16 \times 9$

$d = 144$

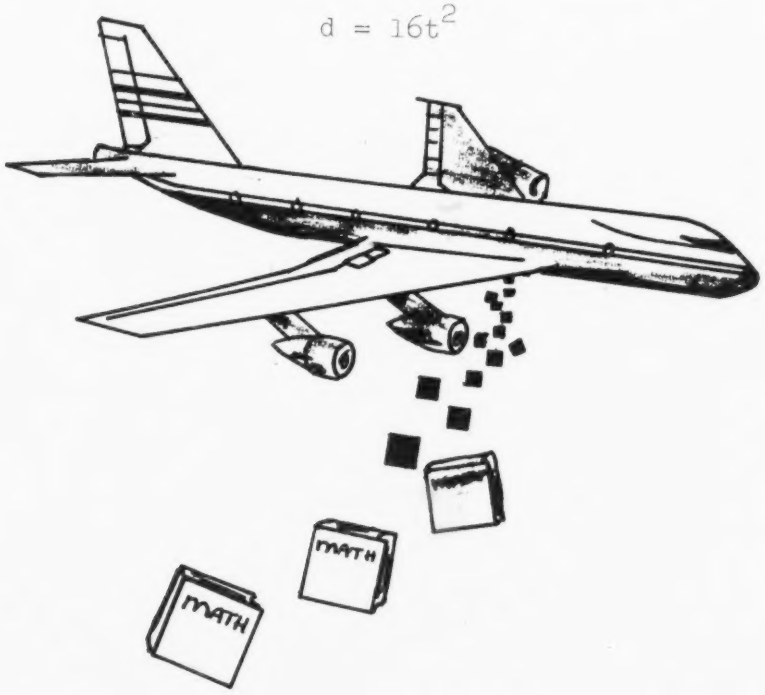
7. Find the height from which an object was dropped. It fell five seconds before hitting the ground.

$$d = 16t^2$$

Notice that in this problem you didn't have to subtract or divide both sides of the equation by anything. All you had to do was multiply: $16 \times 5 \times 5$. This happened because the letter you were to solve for (d) was already all by itself on one side of the equation. This often happens when working with formulas.

8. If a plane drops something and it takes ten seconds for it to hit the ground, what is the altitude of the plane?

$$d = 16t^2$$



60D = rt can be used to find either distance, speed or time if the other two are known.

For example: If D is five miles, and t is six minutes, find r.

$$60D = rt$$
$$60 \times 5 = r \times 6$$
$$\frac{300}{6} = \frac{6r}{6}$$
$$50 = r$$

The speed is 50 mph.

9. Use 60D = rt to find t if D is eight miles and r is 40 mph.

$$60D = rt$$
$$60 \times ____ = ____ \times t$$

Solve for t.

10. Use 60D = rt to find D if r is 20 mph and t is 21 minutes.

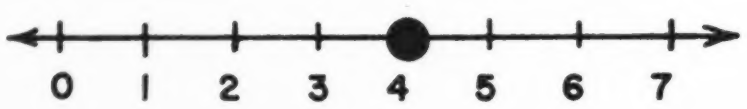
$$60D = rt$$
$$60D = ____ \times ____$$

Solve for D.

SOLUTION PICTURES

Draw a picture of the solution. Example:

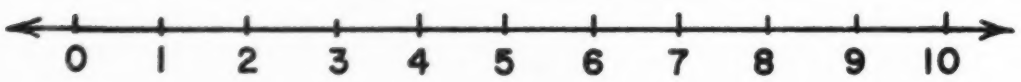
$$\begin{array}{r} X + 5 = 9 \\ \underline{-\ 5 = -5} \\ X = 4 \end{array}$$



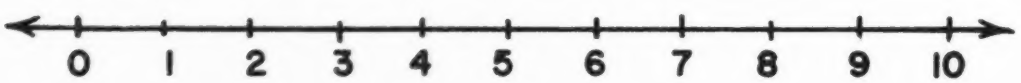
$X = 4$ is the solution because if X is replaced by 4 the sentence $X + 5 = 9$ becomes a true sentence, $4 + 5 = 9$. We show the graph of $X = 4$ as a dot on the number line at 4.

EXERCISES

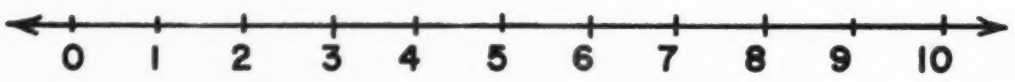
1. $X + 12 = 17$



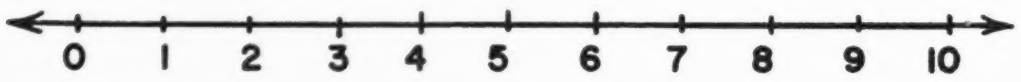
2. $3X = 15$



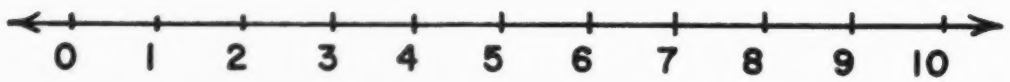
3. $5X = 15$



4. $X + 147 = 152$



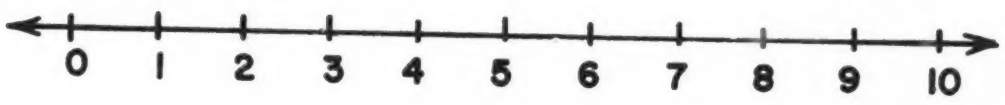
5. $12X = 36$



EXERCISES (continued)

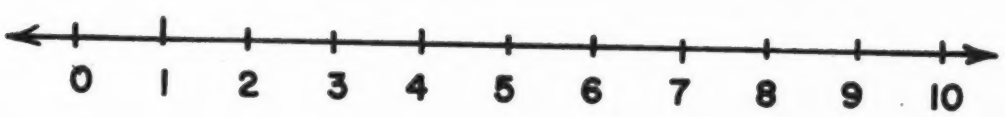
6. $X + 1435 = 1441$

7. $2X + 5 = 13$

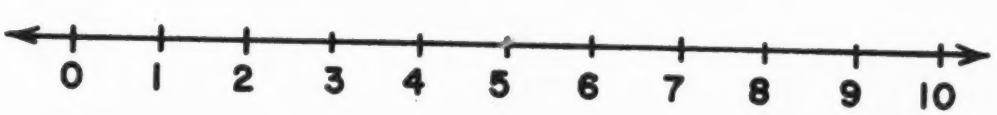


8. $4X + 4 = 16$

9. $5X + 19 = 29$

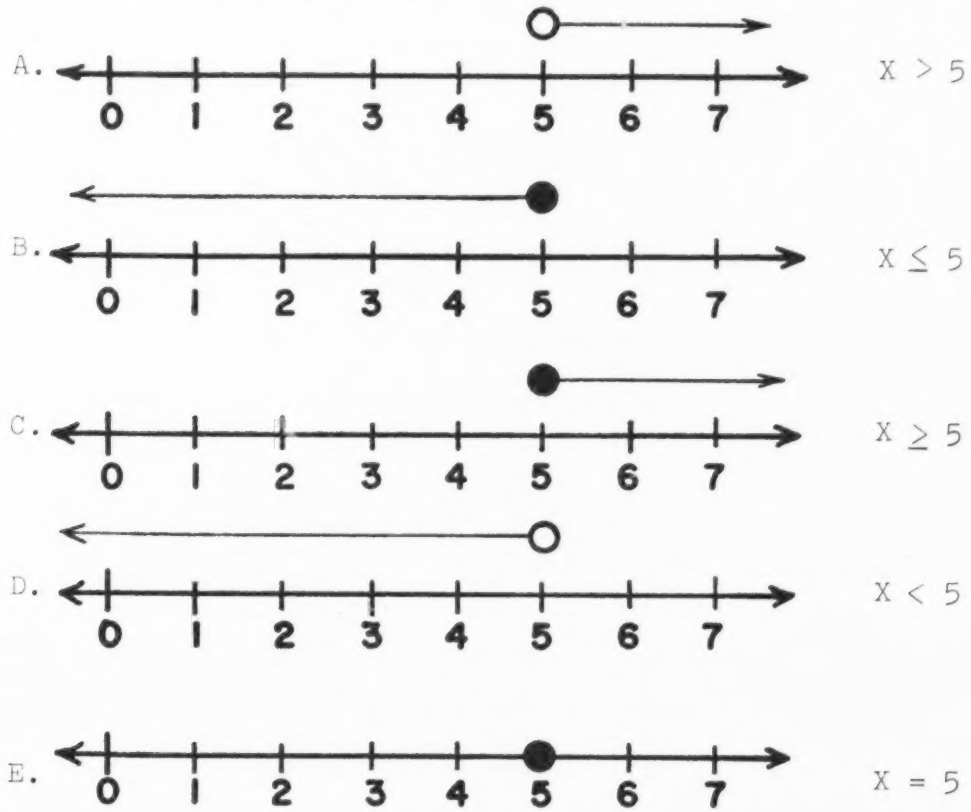


10. $3X + 9 = 21$



PICTURING SOLUTIONS OF NUMBER SENTENCES

The number line is more helpful in picturing solutions of mathematical sentences that have more than one number for a solution. Here are some examples of inequalities using $\geq$, $>$ $<$, and $\leq$ graphed on number lines. As you remember, $\leq$ means less than or equal to. Thus $x \leq 5$ means that any number that replaces the x must be smaller than or equal to 5. Here are some examples of these graphs.

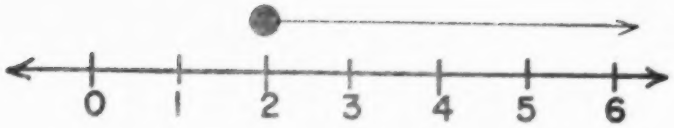


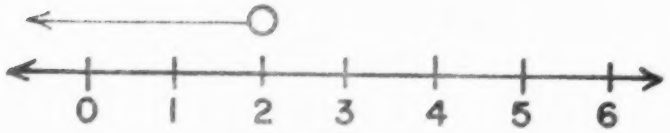
DISCUSSION QUESTIONS

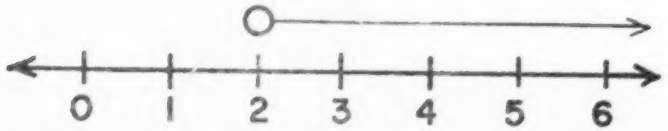
1. Why were the arrows drawn on some of the number lines?
2. Why did the arrows point in different directions?
3. Why do some arrows start with a ● and others start with a ○ ?
4. Why isn't there an arrow in E?

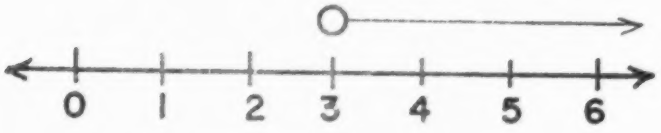
EXERCISES

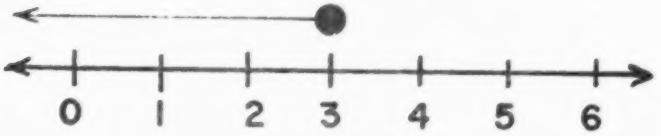
Match the mathematical sentences with their graphs.
The same sentence may be used as many times as necessary.

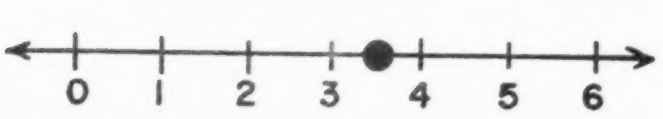
1. 

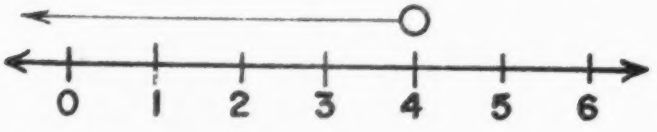
2. 

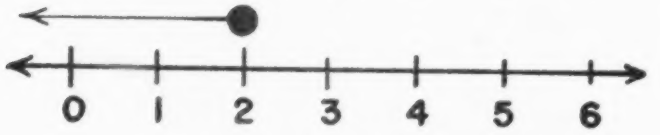
3. 

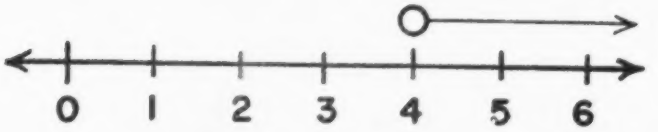
4. 

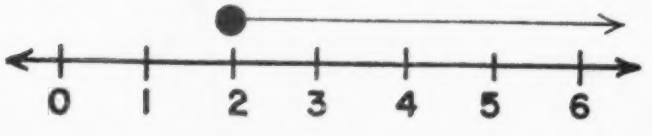
5. 

6. 

7. 

8. 

9. 

10. 

A. $X > 3$

B. $X < 4$

C. $X > 4$

D. $X \leq 2$

E. $X < 2$

F. $X \geq 2$

G. $X \geq 3$

H. $X = 3.5$

I. $X \leq 3$

J. $X > 145$

K. $X \geq 4$

L. $X > 2$

Draw the graph of each of these mathematical sentences on the number line.

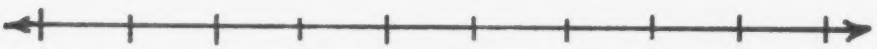
1. $X \leq 3$

2. $X = 7$

3. $X > 5$

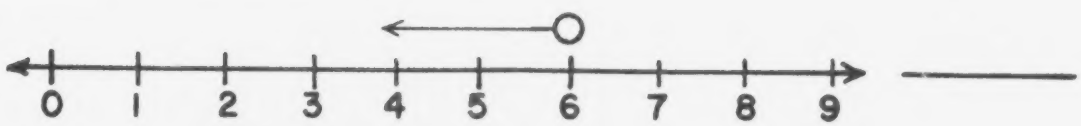
4. $5 < X$

5. $X \geq 2.5$

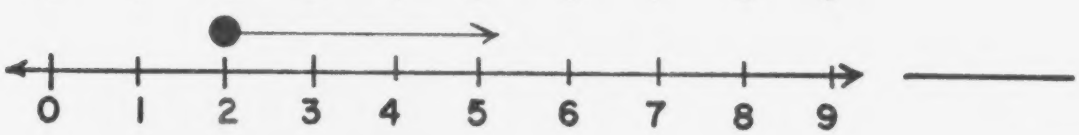


Write the mathematical sentence that is represented by these number line graphs.

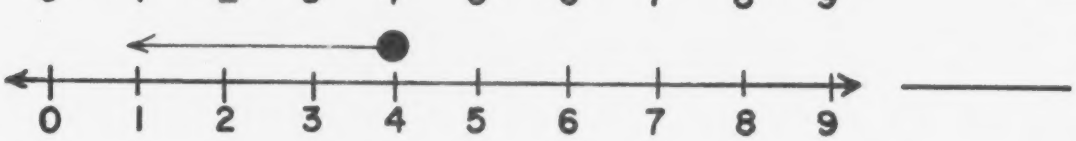
6.



7.

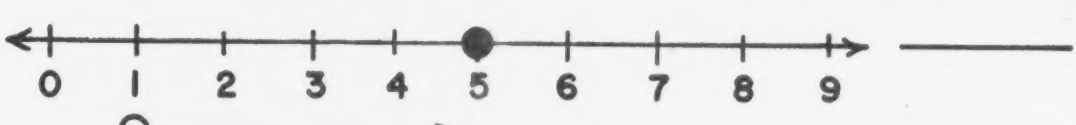


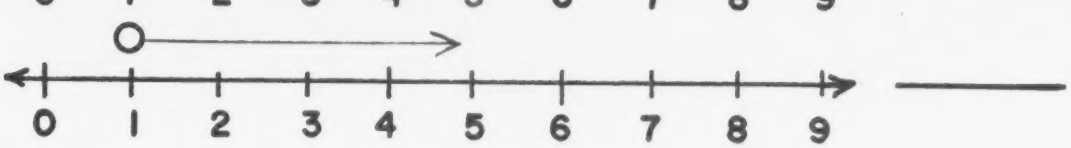
8.



9.

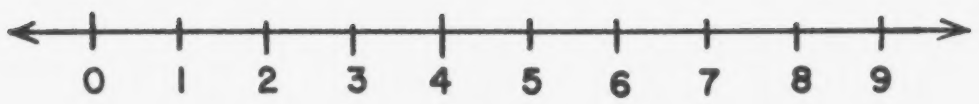
10.



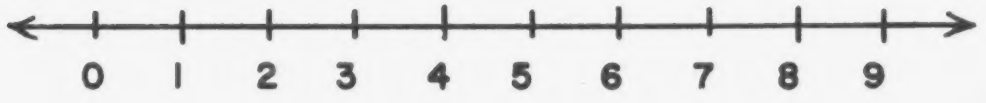


Solve these inequalities and graph the solutions on number lines.

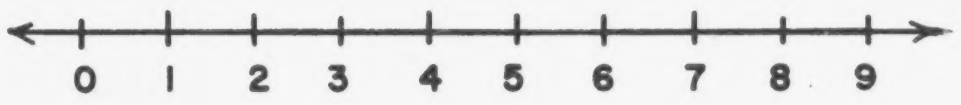
1. $5X > 35$



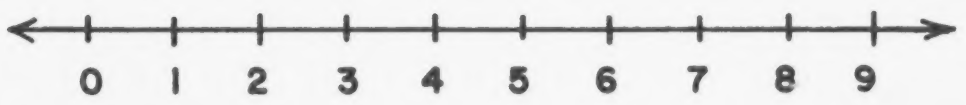
2. $X + 142 \leq 148$



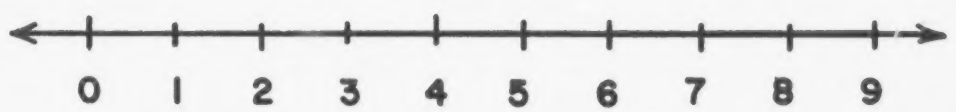
3. $5X + 22 \geq 42$



4. $10X + 17 < 87$

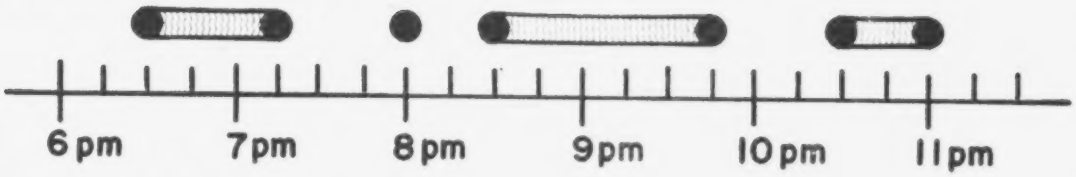


5. $25Y + 23 > 73$



PHONE CALL GRAPHS

Kathy's father is keeping track of the time she talks on the phone. He is an engineer and works with graphs at work, so he made a graph of her calls one Monday evening. This is the graph:



Each unit on the scale represents 15 minutes. The letter t will be used to stand for the times that she was on the phone. The graph shows a dot at 8 p.m. (This would be written $t = 8$.) Kathy answered the phone at 8 o'clock, but hung up right away because she was mad at that boyfriend.

Her longest call was between 8:30 and 9:45. This can be written as an inequality as follows

$$8{:}30 \leq t \leq 9{:}45$$

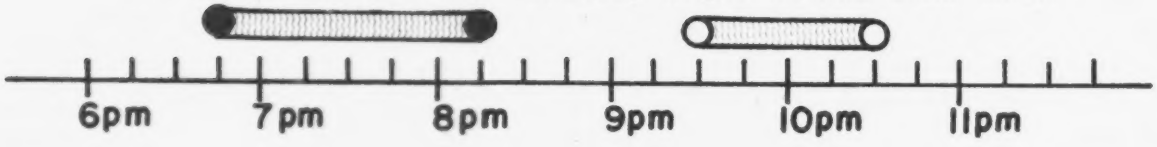
This inequality statement is read "t is greater than or equal to 8:30 and less than or equal to 9:45". Because t stands for those times that she was on the phone, this inequality says that Kathy was on the phone all the time from 8:30 to 9:45. How many minutes was she on the phone during this time span? _____

How many hours is this? _____

As you remember from lesson 8, when the inequality symbols $\leq$ or $\geq$ are used, we graph the endpoints with a "closed dot".

Since the graph of Kathy's telephone conversation has a dot for an endpoint at 8:30 and 9:45, she was talking on the phone at 8:30 and 9:45. If the graph had showed an open circles for endpoints, it would have meant that she started talking right after 8:30 and hung up just before 9:45.

The graph of her calls Tuesday night looked like this:



EXERCISES

1. Describe the time interval she was on the phone on the first call. (For example, "She started at 5:15 P.M. and ended at 6:30 P.M.")
2. Describe the time interval she was on the phone for her second call.
3. Her first call could be written in this form:

   6:45 _____ t _____ 8:15

4. Her second call could be written as:

   9:30 _____ t _____ 10:45

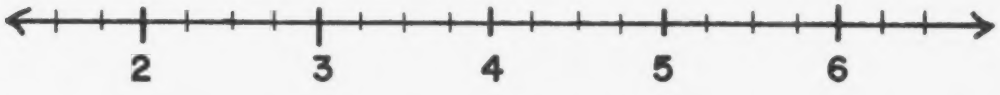
5. How much time did she spend on the phone that night?
6. How much time was left for other things such as studying, watching T.V., listening to records, and washing her hair between 6:00 and 11:00 that night?

7. In making a two-hour lunar orbit, an astronaut is out of touch with the earth for 45 minutes. His radio contact is lost when he is behind the moon.

   The table shown below gives some data on a lunar orbit.

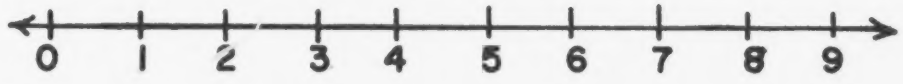
Lunar Orbit	
Time	Information
$t = 3:15$	begin radio blackout
$3:15 \leq t \leq 4:00$	duration of radio blackout
$4:30 \leq t \leq 5:00$	orbit correction
$5:45 \leq t \leq 6:15$	lunar photography

Graph all the times given above on a number line.

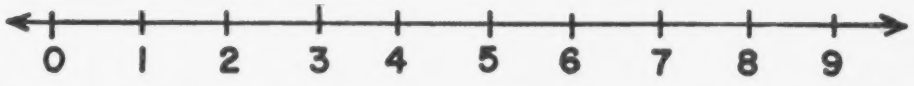


Graph the solution set for each inequality on a number line.

8. $X \geq 1$

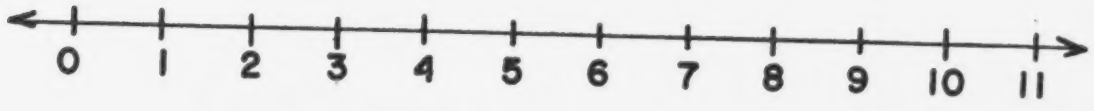


9. $X \leq 8$

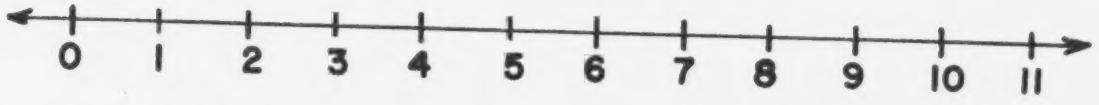


10. $X \geq 3$ and $X < 5.5$

(Hint: Graph <u>each</u> inequality. The solution set is where <u>both</u> are true.)

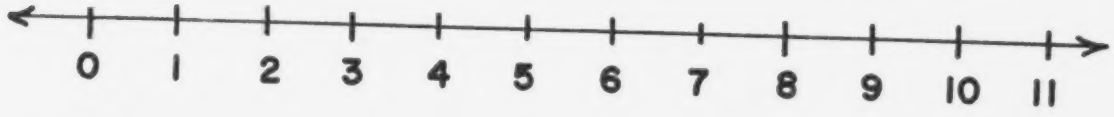


11. $5 < X < 9$



12. $X > 8$ or $X < 3$

(Hint: Graph <u>each</u> inequality. The solution set is all points on <u>either</u> graph.)



WRITING MATHEMATICAL SENTENCES

FORMULAS

DISCUSSION QUESTIONS

1. Above are some common equations. Which ones do you recognize? (Hint: If you don't recognize any of them go back and look at lesson 7.) Tell what these formulas mean.

2. Which of the above formulas do you recognize from your science classes?

3. How many more formulas are there besides the ones listed above?

Formulas are not really very mysterious. Anyone can make one. For example, if your grade in a class was based on your total score on three tests, you might write the formula $S = A + B + C$ to represent your grade.

In this formula S stands for your total score. A stands for your score on the first test. B stands for your score on the second test, and C the score on your third test.

4. What does this formula say is done with your three test scores? _____

Which is easier to write, the description you just wrote or $S = A + B + C$? Actually, a formula is nothing more than a shorter way of stating a rule.

Examples

English Description	Formula
Twice A plus 3 times B plus 5 times C will give X amount.	$2A + 3B + 5C = X$
Centigrade temperature may be found by subtracting 32° from the Fahrenheit temperature and multiplying the result by $\frac{5}{9}$.	$C = \frac{5}{9}(F - 32^\circ)$
The distance an object falls is found by squaring the number of seconds it falls and then multiplying by 16.	$d = 16t^2$

EXERCISES: Fill in the blank.

English Description	Mathematical Formula
1. E added to D equals X.	_____
2.	B + C = 6
3. G added to twice C equals 25.	_____
4.	A + B + C = 35
5. The stopping distance of an automobile can be approximated by squaring the speed of the car and multiplying that number by .055.	Stopping distance = $____^{2} \times ____$
6. A knot is a nautical term meaning nautical mile per hour. To find mph you must multiply knots by 1.15.	mph = _____ × _____

7.	$\ell = 1.9w$
8.	$C = \pi d$
9. One formula used in electric circuits is Volts (E) equals Current (I) multiplied by Resistance (R).	E =
10. Another electrical formula is watts (W) equals amperes (I) multiplied by voltage (E).	W =

Formulas are meant to help people solve mathematical problems. They are actually a set of instructions. For example, in #2 on page 57,

B + C = 6. If B = 5, find C.

If we substitute the number 5 for the letter B, the equation becomes 5 + C = 6. As you remember from working with the balance beam, all you have to do to find C is subtract 5 from each side of the equation.

$$\begin{aligned} B + C &= 6 \\ 5 + C &= 6 \\ -5 \quad &= -5 \\ \hline C &= 1 \end{aligned}$$

11. Using the formula in exercise # 2, find B if C = 4.

12. Using the same formula, find the value of C if B = 3.5.

13. Using the formula you wrote in exercise 3, find the value of G if C has a value of 10.

14. Using formula 5 on page 57, find the stopping distance of a car traveling 30 mph.

15. Find the stopping distance of a car traveling 60 mph.

16. If an ocean liner has a top speed of 40 knots, how many mph would this be?

17. The formula given in exercise # 7 actually gives the relation between the length and width of a United States flag. If a flag has a width of 20 units, find its length.

18. Use the formula in #8 to find C if d is 20. (Use 3.14 or $3\frac{1}{7}$ as an approximation for π.)

19. Using the formula you wrote for #9, find the voltage in an electrical circuit if the current is 10 amperes and the resistance is 17 ohms.

20. Using the formula in # 10, find the number of watts used by an electric frying pan if it uses 10 amps of electricity at 120 volts.

21. Find the stopping distance of these three cars at a dragstrip. At the end of a quarter-mile the first car has reached a speed of 100 mph the second car is going 120 mph and the third car is going 140 mph.

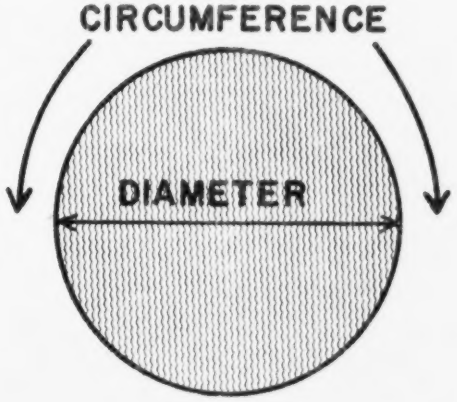
USING MATHEMATICAL SENTENCES

PROJECT #1 Instructions: Gather together several round objects such as tin cans, wastebaskets, paper cups, circles cut out of cardboard, etc. Also you will need a flexible tape measure.

Make a table like this one:

Description of object	Circumference	Diameter	Value of $\frac{C}{D}$

Using a flexible tape, measure the circumference (distance around the object) and the diameter (distance across the object at its widest point).



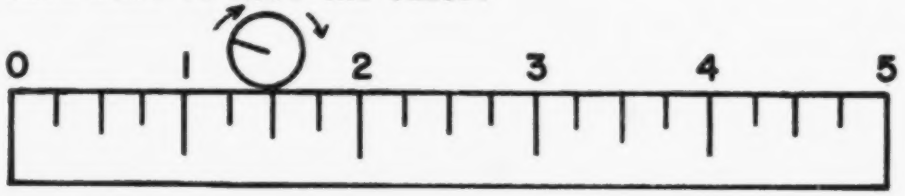
Write down these two measurements for each object in your table. Then, using the formula $C = \pi D$, find an approximation for π.
Example: if C measured 31 inches and D measured 10 inches the formula would be $31 = \pi 10$.

$$\frac{31}{10} = \frac{\pi \times 10}{10}$$

$$3.1 = \pi$$

PROJECT #2

You can measure the distance around a round object by rolling it along a ruler or yardstick. To do this you must make a mark on the edge of the circle. Start with this mark at the zero mark on the ruler. Then roll the object until the mark comes back to meet the ruler.



If you knew the distance a wheel traveled making one complete turn, what would be the distance traveled in two complete turns? _____ How about four complete turns? _____ How far would the circle travel in ten turns? _____ How could you use a circular object to measure distance? _____

An automobile odometer uses this principle to measure the number of miles a car travels. The odometer is connected to the rear wheels on most cars and counts the number of turns of the wheels. The odometer is geared so that turns of the wheel are converted to miles.

DISCUSSION QUESTIONS

How would these things affect the accuracy of a car odometer?

1. Driving fast for a while and then driving slow.
2. Changing to a much larger or smaller tire.

3. Spinning the tires on ice or snow.
4. A hot rodder leaving a patch of rubber.
5. Driving slow, then fast, then slow.
6. A car backing up a great distance.
7. Leaving the engine idling for several hours with the transmission in neutral.
8. Coasting down hills with the engine turned off.
9. A wrecker towing the car with the rear wheels lifted off the highway.

MEASURING BY ROLLING

Any round object can measure distance. The formula is

$$D = nc$$

D stands for the distance the object is rolled. n stands for the number of turns the object rolled. c stands for the circumference or distance around the object.

Example #1 If a wheel with a circumference of six feet makes eight complete turns, what distance does it travel?

$$D = nc$$
$$D = 8 \times 6 \text{ feet}$$
$$D = 48 \text{ feet}$$

Example #2 If a wheel has a circumference of five feet, how many turns does it make in traveling a mile? (Don't forget a mile is 5280 feet.)

$$D = nc$$
$$5280 = n \times 5$$
$$\frac{5280}{5} = \frac{5n}{5}$$
$$1056 = n$$

The wheel must make 1056 turns to travel a mile.

Part A Use a wheel to measure some distances around your classroom or school. If possible measure the same distance several times using different size wheels. Always use the formula D = nc.
Fill in the table below.

What was measured	Circumference of wheel	Number of turns	Distance

Part B Use the formula D = nc to solve these problems.

1. How many turns does a boat trailer tire make to travel a mile (5280 feet) if its circumference is four feet?
2. How many turns does an auto tire make in a mile if its circumference is seven feet?
3. If a tire turns 660 times and has a circumference of eight feet, what distance does it travel?
4. If a go-cart tire makes 25 turns in 60 feet, what is its circumference?

5. If a tire has a circumference of 54 inches and turns 16 times, what distance does it travel?

PROJECT #3 Another way to measure distance is by using the formula D = rt.

D stands for distance.

r stands for the rate of travel.

t stands for the time.

If the rate is given in feet per second then the time must be measured in seconds.

If r is given in feet per minute then t must be measured in minutes.

If r is given in feet per hour then t must be given in hours.

> 60D = rt is a formula you have used before. It is used when r is given in miles per hour and t is given in minutes.

To use either D = rt or 60D = rt to find the distance you must know the rate of travel.

First Step: Find your rate of walking. Have someone time you while you walk off a distance you know (such as the width of your classroom). The timing should be done in seconds.

D (the distance you walked) is _____.

t (the number of seconds it took you) is _____.

With these two values find r.

Second Step: using D = rt

_____ = r _____

_____ = r

Thus your rate of walking is ___ feet per second.

Third Step: Now you're ready to measure other distances. For example, to measure the length of your school hall, time how long it takes you to walk this distance. Try to walk at the same speed you did in the first step of the project.

r (your rate of walking) is _____

t (number of seconds) is _____

D = rt

D = _____ × _____

D = _____

Measure four other distances around your school using this method. Fill in the table and show the mathematics you had to do to get the answer.

D = rt

From what point to what point did you measure?	r (your rate)	t (number of seconds it took you)	D (distance)
Example: one end of hall to the other	5 ft. per second	45 sec.	225 ft.
1.			
2.			
3.			
4.			
5.			

PROJECT #4 Temperature Conversion

For this experiment you need two thermometers, one reading in Fahrenheit and the other in Centigrade. Also you need a glass of water and ice cubes. The group working on this project is divided up into two parts. One part reads the Fahrenheit thermometer and converts these readings into Centigrade degrees. The other students do just the opposite. They record the readings on the Centigrade thermometer and convert them into Fahrenheit degrees. The two groups must take their readings at the same time.

Procedure: The two thermometers are placed in a glass of ice water until the mercury stops dropping. Both groups record the reading of their thermometer at this time. Then the thermometers are removed from the ice water and placed near heat (such as a heat register or the sunlight). Every 30 seconds each group takes another reading on their thermometer.

Each group would fill in one of the tables on the next two pages.

For the group reading the thermometer in Fahrenheit degrees, the formula to use is

$$C = \frac{5}{9}(F - 32)$$

For example, if the temperature read was 42° F., this formula becomes

$$C = \frac{5}{9} \times (42-32)$$

$$C = \frac{5}{9} \times (10)$$

$$C = \frac{5}{9} \times 10 = \frac{50}{9} = 5\frac{5}{9}$$

Then 42° F. must be the same as $5\frac{5}{9}^{\circ}$ C.

Their table is

Time	Degrees Fahrenheit	What this would be in Centigrade degrees

For the group reading the thermometer in Centigrade degrees, the formula is

$$F = \frac{9}{5} C + 32$$

For example, if the temperature is 6° C., the formula becomes

$$F = \frac{9}{5} \times 6 + 32$$

$$F = \frac{54}{5} + 32$$

$$F = 10\ \frac{4}{5} + 32$$

$$F = 42\ \frac{4}{5}^{\circ}$$

Thus 6° C. is the same temperature as $42\ \frac{4}{5}^{\circ}$ F.

The table is

Time	Degrees Centigrade	Degrees Fahrenheit

PROJECT #5 The formula: class rating = $\frac{\text{weight}}{\text{H.P.}}$ is very important at the dragstrip. The National Hot Rod Association places stock cars in various classes according to this formula. To find the class rating, the title weight of the car is divided by its advertised horsepower. Then a table gives the class in which the car will race.

For example, consider these three cars:

A. A 3000 lbs. compact car with a 200 H.P. engine.
B. A 3750 lbs. intermediate car with a 250 H.P. engine.
C. A 4500 lbs. luxury car with a 300 H.P. engine.

The formula for Car A would be:

$$\text{Class rating} = \frac{3000 \text{ lbs.}}{200 \text{ H.P.}} \qquad 200 \overline{)3000}^{\,15}$$

Class rating = 15 lbs. per H.P.

Car B: $$\text{Class rating} = \frac{3750 \text{ lbs.}}{250 \text{ H.P.}} \qquad 250 \overline{)3750}^{\,15}$$

Class rating = 15 lbs. per H.P.

Car C: $$\text{Class rating} = \frac{4500 \text{ lbs.}}{300 \text{ H.P.}} \qquad 300 \overline{)4500}^{\,15}$$

Class rating = 15 lbs. per H.P.

Because all three cars have the same class rating (15) they would all race in the same class. According to the table at the top of page 65, what class would this be? Do you think it would be a fair race?

Here is part of the table the National Hot Rod Association uses to classify stock cars.

A/S-7.50 to 7.99
B/S-8.00 to 8.49
C/S-8.50 to 8.99
D/S-9.00 to 9.49
E/S-9.50 to 9.99
F/S-10.00 to 10.49
G/S-10.50 to 10.99
H/S-11.00 to 11.49
I/S-11.50 to 11.99
J/S-12.00 to 12.49
K/S-12.50 to 12.99
L/S-13.00 to 13.99
M/S-14.00 to 14.99
N/S-15.00 to 15.99
O/S-16.00 to 16.99
P/S-17.00 to 18.99
Q/S-19.00 to 20.99
R/S-21.00 to 22.99
T/S-23.00 to 24.99
U/S-25.00 to 26.99
V/S-27.00 or more

Part A Classify these ten stock cars into their proper class:

1. A 3500 lb car with a 350 H.P. engine.
2. A compact weighing 2475 lbs. with a 200 H.P. engine.
3. A 4000 lb. sedan with a 200 H.P. engine.
4. A 3700 lb. hardtop with a 425 H.P. Hemi engine.
5. A 3240 lb. compact convertible with a 180 H.P. engine.
6. A 2880 lb. economy car with a 120 H.P. engine.
7. A 4100 lb. luxury sedan with a 325 H.P. engine.
8. A 3000 lb. car with a 375 H.P. engine.
9. A 3155 lb. hardtop with a 250 H.P. engine.
10. A 2400 lb. Model A with a 40 H.P. engine.

Part B Find the weight and horsepower of several cars and classify them according to NHRA rules. For each car give the make and model, title weight, horsepower, and class it would run in at the dragstrip.

PROJECT #6 Here are some other formulas. Make up your own exercises.

#1 $W = 5.5(H - 60) + 110$

This formula is thought by some people to give the proper weight of a person.

W stands for weight in pounds.
H stands for height in inches.

Example: Find the weight of a person who is 5 feet 10 inches tall.

5 feet 10 inches equals 70 inches $(5 \times 12 + 10)$
$W = 5.5\ (70 - 60) + 110$
$W = 5.5\ (10) + 110$
$W = 5.5 \times 10 + 110$
$W = 55 + 110$
$W = 165$ lbs.

Thus the formula tells us that this person should weigh 165 lbs.

#2 Imperial gallons $\times$ 1.2 = U.S. gallons
U.S. gallon $\times$ .8327 = Imperial gallon

#3 Batting Average $= \frac{H}{B}$

H = number of hits
B = official times at bat
The division is carried out correct to three decimal places.

Example: A batter has 27 hits out of 72 official times at bat.

Batting Average $= \frac{27}{72}$ $\qquad 72\overline{)27.000}$ $\quad .375$

Thus, his batting average is .375.

#4 $ERA = \frac{ER}{I} \times 9$

The earned Run Average is a way of comparing the abilities of baseball pitchers.

The formula used is $ERA = \frac{ER}{I} \times 9$

ER is the number of earned runs the pitcher has allowed.

I stands for the number of innings the pitcher has pitched.

Example: A pitcher has allowed 15 earned runs in 45 innings.

$$ERA = \frac{15}{45} \times 9$$

$$ERA = \frac{1}{3} \times 9$$

$$ERA = 3.00$$